A collaborative inquiry to promote pedagogical knowledge of mathematics in practice

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The present study attempts to report a collaborative cycle of professional development in teaching elementary school mathematics through lesson study. It explores a practice of lesson study conducted by teachers aiming to improve their knowledge of pedagogy. The study adopts an ethnographic approach to examine how collaborative teaching within an adapted lesson study framework might change the teaching-learning process. More explicitly, the present research looks at how lesson study influences teaching mathematics and how it helps the teachers learn from their peers in a discursive school-based setting. The study suggests that teachers need to aim high when dealing with students, and use more daily life situations in their math problems. It also particularly reveals that lesson study could have the potential to help teachers promote their teaching and boost students' learning. Furthermore, it might also be used as an effective alternative to traditional professional development programs.

Introduction

Lesson study is a chain of planning, teaching, revising, and re-teaching a lesson collaboratively (Lewis, 2002). It has a long history in Japan, and has been practiced by teachers in the US, some European countries, Iran, and East Asia for quite some years. However, it was with the introduction of *The Teaching Gap* by Stigler and Hiebert (1999) that the world came to know how Japanese teachers practise teaching mathematics in their classrooms and how such a practice improved students' performance in the Third International Mathematics and Science Study (TIMSS). It was not the first time, however, that Stigler realised the cultural differences between Japanese and American teaching methods. Describing his first trip to Japan in the foreword of *Lesson study: A Japanese approach to improving mathematics teaching and learning* (Fernandez & Yoshida, 2004), he explains that it is not the Japanese way of "cookie wrapping" as an "isolated practice" which differs from that of the Americans, "but just another way that Japanese approach many things, including teaching and learning" (p. ix). With this difference in our minds as well as having the advantage of two of the author's long experiences in a Japanese educational context, we decided to implement lesson study in Iranian schools. We intended to benefit from lesson study as a means of collaborative inquiry as well as a framework for teachers' professional relationships, reflective practice and learning from each other (White, Jaworski, Agudelo-Valderrama, & Gooya, 2013).

The implementation of lesson study and the way Iranian teachers conceived of it were not as easy as the framework suggested. Such a barrier is a common concern for some practitioners elsewhere as well, as Lewis, Perry, Hurd, and O'Connell report "how to do it [lesson study] was much less clear" (2006, p. 273). Iranian teachers thought that they could

adopt the lesson study stages and carry them out without considering that all these processes had, by nature, cultural ingredients which made lesson study different from context to context. Lesson study "fosters a culture in which, as one put it, 'you're learning. You don't know everything.' So teachers feel safe revealing gaps in their knowledge" (Lewis et al, 2006). Hence, lesson study requires a culture of openness, collaboration and self-reflection; however, there are studies in a number of Iranian educational contexts which show that individual work is more common than group work (Moghaddam, & Sarkar Arani, 2006). Having said this, the teachers and the authors had to choose either of the two following perspectives: 1) They could wait until these characteristics come true as a result of a series of professional development programs and then implement lesson study, or 2) They could implement lesson study and hope that these characteristics follow lesson study as outcomes since lesson study has the potential to develop these traits- some traits can be developed by practice-, and we think that there is no better way to learn collaboration than practicing collaborating. Either one of these perspectives could lead to different strategies, and consequently different directions. We chose the second perspective, as we believed that lesson study had the potential to create its prerequisites as it was being implemented. In this study, the authors tried to: 1) realise how the teachers of an elementary school perceive lesson study; and 2) what professional development potentials lesson study could have for them.

Rationale for lesson study in Iran

Iran has a high youth population and about 17% of the total population are school-age students (Statistics and ICT Center, 2013). Such a high rate could be partially accounted for due to a policy that started about three decades ago. In the early 1980s, a change in the state's birth control policy caused the population to grow rapidly, and consequently there appeared classrooms jam packed with students in less than a decade. As far as meeting the needs of these classrooms and providing a sufficient number of teachers were concerned, those who had either 8 or 10 years of schooling were allowed to attend Teacher Training Centers to become teachers. They had to spend 4 or 2 years respectively before they could teach at elementary schools. Because of such a policy most of the teachers found a way to enter the Ministry of Education without having university education. Even though there are currently no Teacher Training Centers and they have been replaced with Teacher Training Universities, which provide a four-year course after graduation from high school, there are still teachers who graduated from those Teacher Training Centers without pursuing any university education. Hence, still a considerable number of teachers only have a high school diploma without any university experience (Sarkar Arani, 2003; Statistics and ICT Center, 2013). As a result, there is a crucial need for professional development.

Although the Ministry of Education has developed several types of pre-service and in-service teacher training programs to improve the qualifications of teachers, they have failed to be as effective as they are expected to be. It is because the former pre-service programs consisted of two-year highly theoretical courses, and pre-service teachers only attended schools once a week in their second year. Pre-service teachers usually watched

the teachers teach the class and they were occasionally allowed to teach the course, this experience was called Karvarzi (practicum). The in-service training programs, on the other hand, are conducted either as an independent reading course or as on-site-based courses, or a combination of the two. These programs are planned and implemented by the Ministry of Education. The Ministry rarely considers the needs of individual teachers or the problems they face in their educational contexts. Therefore, instead of taking a bottom-up approach to consider the practical needs of teachers in the classroom, the in-service training program developers take top-down approach to prepare and conduct professional development plans (Jadidi, & Bagheri, 2014; Shirazi, Bagheri, Sadighi, & Yarmohammadi, 2013). Such a reverse approach might lead teachers to being "passive recipients" of knowledge, most of whom may "ignore" training programs or may not be willing to implement what they have been offered without taking into account their pedagogical needs (Duffy, 2014; Lee, 2008, p. 1115).

In recent years, however, there have been changes with respect to professional development through in-service teacher training programs. A few years ago, Iranian teachers were introduced to action research (Elliot, 1991) and more recently to lesson study. Although action research was highly embraced by Iranian teachers, lesson study needs serious consideration to find its place so that teachers can benefit from its professional development opportunities. In order to see how lesson study might give an insight into elementary school teachers' practice, the present study was proposed to the Iranian Ministry of Education, planned and carried out intending to provide teachers with a means of bottom-up professional development. Since lesson study originated in Japan and, since then, it has been adopted in various countries, including Iran, we will give a brief comparison of professional development in Iranian schools with those of Japanese before introducing the school at which our study was conducted.

Unlike Japanese teachers' professional development, where it takes place in classrooms through reflecting on their own educational practices and during the lesson study cycle of Plan, Do, Check and Act (PDCA) (Sarkar Arani, 2006), Iranian teachers' professional development is not done in classrooms. Based on two of the authors' experience with teachers in Iran, there is the least opportunity in schools for teachers to learn from each other. The collaborative PDCA cycle rarely takes place in professional development programs in Iran since the professional development programs usually rely on knowledge-based tests only, and teachers tend to work more individually than collaboratively. As Sarkar Arani (2006; 2015) describes, while Japanese teachers "improve their teaching in practice … thus emphasising learning by doing" (p. 47), in-service teacher training programs in Iran require teachers to study some designated books to take the test in order to receive professional development certificates in a specific course. Yet another difference is that when it comes to teacher evaluation, Iranian teachers are evaluated by the principal only, and receive feedback on the aspects in need of improvement. In Japan, such feedback comes from self-reflection and peers as well as from the principal. These distinctions are only a few among many more differences; consequently we expected to see some difficulties in conducting lesson study in the Iranian context.

The lesson study case which is being reported here was implemented by the teachers of a private elementary school in Tehran, Iran's capital city. The principal of the school explains her intention of establishing the school as follows:

> I established this school in order to realise, in the freedom of a private school, how possible it is to carry out what I had learned through all my teaching-related schooling - five years in Teacher Training Centers; four years at the university as a BEd student; and attending in-service teacher training programs for more than 1000 hours. Through all these years, many scholars and teachers helped me with the school management, but the results were only students' high marks and their parents' ostensible happiness. … students were not interested in their school, and teachers looked like strangers to their colleagues, and I alone was looking for what I had lost.

The authors believe that private schools have more freedom to maneuver. In juxtaposition to public schools, they have more authority to implement their own plans. Therefore, we thought, to start with, it would be less difficult to carry out lesson study in a private school. With that being said, as the principal describes the educational situation of the school, the expected outcomes of schooling were not, educationally speaking, satisfactory even though the school was a private one supported financially by parents and educationally by scholars and trained teachers. We think that high marks do not necessarily represent high quality of educational outcomes, especially if these marks are the result of tests aiming at students' memorisations.

With the introduction of lesson study, the teachers were asked to actively engage in a collaborative process to improve their teaching. Teachers chose second grade mathematics to prepare a lesson plan and worked on it within the lesson study framework. In briefing these teachers, we, as the authors of the present study, emphasised that in order to use such a framework, they had to work with each other as a team in their workplace. Such collaboration was crucial, in part because of the nature of lesson study that requires group work.

Research context

As part of a School Enrichment Project, we proposed the Japanese model of lesson study and adapted it for implementation within a teacher quality improvement program in Iran. With our study exploring mathematics teaching at the elementary school level, such experience reflects how lesson study was conducted in an Iranian educational context. This project included programs such as teacher training, curriculum development, school improvement, raising the leadership ability of principals and reforming teachers' professional development and community relationships. The project prompted the Organisation for Educational Research and Planning, a section of the Iranian Ministry of Education, to utilise it in 38 elementary schools in Tehran (19 districts - 19 boys' schools and 19 girls' schools).

Originally, lesson study is a cycle to improve teaching, which concentrates on planning, observing, reflecting and revising the teaching-learning process. The cycle provides teachers with a learning experience which is directly connected to the classroom, so it can

help teachers understand teaching-learning practices and ways of improvement. This approach has multiple functions, benefits and outcomes. On the one hand, it is a means of re-conceptualising professional improvement and bringing teacher development programs from colleges and universities to schools (Hodge, 2014) - which could be a more effective place to improve their teaching performance (Rock & Wilson, 2005). In the words of Little, schools are more likely to "play a powerful, deliberate and consequential role in teachers' learning" (Little, 2012, p. 23). It may also change teachers' roles from teachers vs. learners to teachers as learners and researchers (Anderson, Bobis, & Way, 2008; Fernandez & Chokshi, 2002; Krainer, 2014). On the other hand, such a cycle has the potential to develop and enrich human relationships among teachers, who used to work individually, and changes teacher-student relationships, which used to be on a one-teacher-per-class basis (Darling-Hammond, 2003).

Another, yet important, characteristic of effective professional development programs is that they consider the teachers' interests and needs (Stover, Kissel, Haag, & Shoniker, 2011). In the lesson study approach, teachers' interests are, in fact, in the core of professional development activities (Lieberman, 1995). Contrary to the centralised in-service teacher training programs of the Ministry of Education, which are less likely to consider individual teacher's interests, lesson study respects them in terms of providing opportunities to discuss the problems they face in their classrooms. Lesson study has a unique characteristic of instant feedback which does not require teachers to wait for a year in order to make instructional changes. Hence, as collaborating peers, teachers can review each other's work and share immediate feedback after conducting classroom observations, and assist one another in solving problems they face in authentic contexts (Rimpola, 2011). It may also prepare the ground for them to practice their thoughts in a real situation, distinctive features that Fernandez (2005) calls "temporal" and "concrete" (p. 283). The present research adapted lesson study framework and concentrated on professional development as one of the functions which lesson study can perform to overcome the low level of professional skills mentioned as the research problem.

Research method

The participants of the study consisted of five teachers, the principal, and 18 grade two students of a private elementary school. They were all female as the schools are segregated in Iran. The teacher who volunteered to teach the lesson was a grade two teacher and had 14 years of teaching experience.

The data collection involved classroom observations, ethnographic field notes, lesson artifacts prepared by the participants, interviews and videotapes. The authors used the Japanese lesson study approach to analyse, understand and examine the lesson. This approach can be described in the following steps: first, all videotapes were transcribed and viewed several times with the transcripts. Then the lesson manuscript was interpreted taking into consideration the field notes and lesson artifacts to make better sense of the teacher's practice and the teacher-students communication. Finally, the lesson analysis was concluded providing a number of suggestions in order to improve teaching and professional development.

All the teachers from grade 1 to 5 participated in this process. The principal also was present in the sessions which needed coordination. Such coordination included arrangements to involve free students in other educational activities. The role of the authors as researchers was to document the process, ask insightful questions, prompt reflection, and remind the participant teachers to critique teaching, not the teacher.

Lesson study in practice

Teachers conducted three stages of lesson study as we observed them involve in a discursive cycle of professional development. These stages include: Plan, Do and Check. Originally, as mentioned earlier, lesson study has four stages (Kuno, 2015; Moghaddam, & Arani, 2006): Plan, Do, Check and Act (PDCA). The reason that the participants in this study carried out only three stages out of four was that (1) the main purpose of the study was to find out how lesson study might have the potential to help professional development, and (2) the participant teachers complained that they did not have time to redo (Act) the lesson which they had already taught because of time restrictions and the textbook requirements they had to cover by the end of the school year.

Based on this adapted three-stage lesson study framework, participant teachers started preparing a plan for their collaboration. Since cooperation between the school staff (namely the principal) and teachers was critically important in conducting the improvement plan, teachers and the principal had several meetings to discuss and set the stage in order to conduct their plan. After ensuring that there was sufficient understanding of the collaborative process, they started planning for lesson implementation.

Lesson plan

The first step was to Plan for lesson study. The participant teachers reviewed the mathematics textbooks of all five grades and selected Grade Two as their focus. The desired concept was Addition of two one-digit numbers. As we will describe it later in this section, teachers planned to teach this concept using the set model by means of a real-world problem. Calling this type or problems "story-problem problem", Sowder (1995) believes that there is only one type of setting linked to addition: "groups or amounts are put together, either physically or conceptually" (p. 129).

After discussing about the content of the selected course, teachers developed the following learning objectives for it:

1. Based on their previous learning, students should know the concept of numbers, addition, the addition sign and the equation sign.
2. They should be able to write an equation for a problem.
3. They should be able to read an equation and understand its meaning.
4. They should be able to apply their understanding of addition to real-life problems and solve them using addition.

In order to conduct the course and reach the aforementioned learning objectives, the teachers designed the following lesson plan.

Table 1: Lesson plan

Section	Time	Activities
1	5	Proposing a problem and stimulating students to think about it
2	3	Providing opportunities for students to think and find solutions
3	7	Students will propose solutions, and the teacher will write them down on the board
4	10	Discussing the solutions
5	10	Providing students with new problems and opportunities to find solutions
6	5	Conclusion
7	3	Evaluation

For the purpose of involving students and attracting their attention, the volunteer-grade-two teacher should start the session with giving the group members walnuts that she brings to class. Then she provided each group of students with an addition problem. The problem that she had designed to give to the students was how they could increase the number of walnuts they were given. The teacher had planned to give them three minutes to think about the problem and prepare their answers. The students were assigned in small groups to start discussing the problem and the potential solutions. Then the teacher asked a member of each group to propose their solutions. The teacher was supposed to write down their solutions on the board and start asking for the pros and cons of each solution while she had to try to give students hints whenever they had difficulty coming up with appropriate responses. Then the teacher should bring the groups together so that they could discuss real-life representations of the problem. After the group gave the teacher their real-life examples about the problem in question, the teacher will summarise the discussions. At this point the teacher should ask students to draw pictures to show similar examples and explain what they might have learned. In order to provide the students with more learning opportunities, the teacher was supposed to propose a number of related problems, do final assessment and ask the students to do textbook problems to stabilise learning.

The second step was to Do the plan which was collaboratively prepared. One of the teachers agreed to conduct the lesson plan while the rest of the team participated in the class and actively observed the whole session. They made notes of what was happening in the classroom and each of them was to observe a particular point in the lesson. For example, one teacher was responsible to make notes of the volunteer teacher's classroom management skills; another teacher was to observe and write down how the teacher was engaging students; and yet another one was looking at the interaction among students and their individual or collaborative activities.

Lesson observations

The teacher started the class by giving each and every group of students a card with a number written on it. Then, she asked the students to read the number and pick the same

number of walnuts. Now that every group had picked up their number of walnuts, the teacher asked a question to provoke students' thinking. Here is how the teacher exposed the essential question to draw the students' attention to the concept she was teaching:

> Teacher: Now, I have a question. Listen everybody. I want you to think and answer my question. Each group with the help of its members should think and let me know how we can increase the number of our walnuts. You have only three minutes.

Students discuss the problem in groups and as soon as they come up with their answers they raise their hands to answer the teacher's question. However, the teacher asks students to keep their answers in their mind until all groups are ready. The following shows how the teacher and students communicated to come up with the right solutions:

> Teacher: Ok, seems that you are ready to let us know your solutions. What is your solution [pointing to one of the groups]?
>
> Student 1: To split them into halves.
>
> Teacher: Ok, so you think we can do it that way? [The teacher writes down the proposed solution on the board]. What do you think [pointing to another group and asking for their solutions]?
>
> Student 2: To split and plant our walnuts.
>
> Teacher: The other group already said one of the solutions, but I write the other one.
>
> Student 3: To smash them.
>
> Student 4: To smash them is the same as splitting into halves.
>
> Teacher: [The teacher writes 'smash them' on the board].
>
> Student 5: To divide them into five pieces.
>
> Student 6: To plant a few walnut trees.
>
> Teacher: [The teacher writes 'to plant walnut trees' on the board].

After the students gave their ideas, the teacher started asking them to reject the incorrect solutions:

> Teacher: Ok, let's see which one of these proposed solutions are correct. The first solution is to split the walnuts into halves. If I have a walnut and split it into halves, do you think it becomes more? Or not? Let's split it to see what happens.
>
> Teacher: [The teacher tries to split one of the walnuts].
>
> Teacher: Now, I split it into halves! Is it really more than it was before?
>
> Some students: Yeeees!

Teacher: Really? Is it two walnuts now?!

Some students: Noooo!

Teacher: What if I put the two halves on each other and keep them together?

Teacher: [The teacher puts them together and shows it to the students again]

Teacher: So! It does not become two. How many is this?

Some students: Only one!

As the teacher-students communications show, the students did not come up with an appropriate answer which the teacher had in her mind. So, the teacher started questioning the students' answers while she did not try to give a direct solution. But students still had difficulty solving the problem. The teacher then decided to give the students more time so that students may find solutions other than what they had already proposed.

Teacher: I think you need more time to think the problem over. Now, work again in groups, think and let me know how you can increase the number of your walnuts.

The teacher waits until students discuss the problem and come up with solutions.

Student 1: Groups can share their walnuts.

Teacher: What does it mean to share walnuts?

Student 1: to put the walnuts together.

Teacher: So, do you mean if we put the walnuts together they increase in number?

Student 1: Yes, my group and another group can put our walnut all together to have more walnuts.

Student 2: [Putting walnuts together] now, there are 8 walnuts. They had 4 and we also had 4.

Teacher: So, what did you do?

Group 4: We have 8 walnuts now.

Teacher: Now, the students in group 4 tell you their story. What happened when you consulted the issue?

Student 2: We consulted and came to the conclusion that if our group and the other group next to us add the walnuts, the number of walnuts increases.

Teacher: Now, draw a picture on the board to show how many walnuts you had before.

Student 2: [Draws 4 walnuts on the board.

Teacher: How many walnuts did the other group have?

Student 2: 4 walnuts.

Teacher: [The teacher asks a member of the other group to draw the walnuts they had on the board].

Teacher: Ok, how many walnuts do we have now?

Student 3: 8 walnuts [she writes '8' on the board].

After doing a few examples of the problems mentioned above, the teacher gave the students assignments to practice both in the classroom and at home.

Reflections on the lesson

Once the lesson was conducted, the teachers got together to Check the implemented plan and discuss the lesson. The teachers analysed, critiqued and evaluated the teaching method, class management, students' interaction, and the teaching materials used by the teacher. The teachers sometimes asked the authors for their advice and feedback on the lesson conducted, but we had decided not to engage in the reflection process and, instead, allow them to discuss and explore it by themselves. Thus, everyone proposed their own ideas and the issues raised were touched upon.

There were a number of problems that teachers discussed at the reflection stage of their lesson study. Some of the points which were raised in the group and reflected on are as follows:

- Proposing the problem was not clear enough so students did not have a clear idea of what the teacher meant.
- Some of the students did not pay enough attention and did not participate in the discussions.
- There were quite a few students who participated in the discussions and the rest were silent.
- The questions and problems in the textbook were overlooked.
- The materials, which were used as teaching aids, were not suitable enough (the picture on the cards were so small that the students at the back were not able to see them clearly).
- It was better if the students were called by their names when they were asked to say their ideas and propose their solutions.
- Before proposing the problem, the students were not ready enough.
- The interaction between the teacher and students was satisfactory.

Findings

There were some incidents in the lesson study that stood out, and we think they are worth being considered in order to improve teaching and learning. With respect to the research

question regarding the teachers' perception of the lesson study, we could state that collaboration and process-orientation were the elements of lesson study that the teachers had rarely experienced before. In the course of teachers reflecting on the lesson plan implementation, not only was the teachers' collaboration important, as it provided them with the opportunity to learn from each other, but also the students' collaboration took place as well — they had the opportunity to work together to solve problems which perhaps would be even more difficult to solve individually. They also had the chance to figure out that mathematics was everywhere in their daily lives and life would be much easier if they learned mathematics and its applications.

In a self-reflection, the teacher who was responsible for the implementation of the plan admitted that the result of the collaborative teaching was informative and thought provoking:

> Today's teaching experience felt more effective and satisfactory than my other professional development experiences. When I saw the class videos and listened to my colleagues, I figured out that there was a lot to learn to improve my teaching. At the checking stage, when my colleagues were analysing my class and raising the problems, I was trying to defend myself, however as we practiced, I learned that they were not criticising me, but the teaching-learning process to help me improve it. … I think that I should find better ways of stimulating students' interests. In the beginning of the class, the problem that I raised led students in a wrong direction and caused them give answers other than I was expecting.

This reflection includes a number of points which depict how the collegial collaboration during the check stage was beneficial. As the teacher reflects, the problem raised by the teacher was problematic itself. In other words, the teacher herself wonders if the proposed problem was the most appropriate task to give students. This implies the teacher might look for other alternatives to include in the lesson plans for future sessions. To have alternatives fosters the teacher's performance when a particular method does not work.

In terms of professional development programs, as mentioned earlier, teachers are required to study text-based courses which are usually far from the participants' needs. Unlike this, lesson study, as this volunteer teacher reflects, was mainly relevant to her shortcomings pertaining to her own teaching. It is in such a situation that teachers could relate to the professional development programs and feel that they are at the center of the courses.

As another reflection, a young teacher wrote her supervisor the following note after the lesson study session:

> I always thought that in mathematics education, what was most important was for us to emphasise comprehension of the correct answer and the formula. We are pressed for time in the classroom and never have enough time to cover everything. My teaching places more emphasis on student achievement and the results of weekly, monthly, and quarterly tests in school. However, through the lesson plan meetings, class observation

> and reflection sessions, I was pondering that we need to place more emphasis on understanding the problem and the process of problem solving. The lesson study, as I got involved in, might have the prospect to help us understand how to improve the learning process of each student.

The point of importance in this reflection is that even though the teacher is aware of the time boundaries and the textbook requirements, she thinks that there could be a way to overcome this when lesson study is implemented. Such feedback reveals the likelihood of perception shift of the teacher's from the output value to the process value. With this being said, we need to mention the fact that Iranian elementary textbooks are too massive, but the time is too short. This issue has frequently been mentioned by the teachers throughout the authors' experience while working with them. However, lesson study might lead the teachers to build new pedagogical knowledge to overcome some barriers which take too much time for a particular subject and save time for other subjects even though focusing on the process rather than the "correct answer" is time consuming.

One of the aspects that we observed in the course of lesson study was that the adapted lesson study framework provided the teachers with the opportunity to develop the pedagogical knowledge of mathematics. We found that the participants worked with each other, planned for their lessons, reflected on their teaching practices, practiced critiquing the teaching not the teacher, and disagreed without seeming disagreeable. They also recognised to focus on students' learning abilities after participating in the lesson study, based upon self-assessment as well as feedback from the colleagues who were observing the class. This reflection might set the stage to help the teacher:

- Understand students' interests and capacity in a mathematics classroom;
- Understand students' interests and provide more time for students to participate in the classroom;
- Think about the ways of helping students work on problems by themselves;
- Improve assumptions about students' abilities of understanding mathematical concepts;
- Develop teacher-student interactions in mathematics classrooms;
- Provide more teaching material for mathematics classroom activities related to the daily life of students.

Discussion

Considering the issues raised by the teachers at the reflection stage, it seems that the teachers have found out some critical problems with the teaching-learning processes. Although the teachers had neglected the importance of problem solving at the planning stage of lesson study, it seems they come to figure out that the notion of real-life problems needs more attention, and students do not master mathematics unless they acquire the most important learning objective — problem solving and applying the mathematical rules to the real-life situations (Braun, 2014; Mullis, Martin, Ruddock, O'Sullivan, & Preuschoff, 2009; OECD, 2013; Sahlberg, 2011). As Sarwadi and Shahrill put it,

> Teaching the students only procedural skills will impair learning in the classroom and will not equip students well with the necessary skills mathematically for the future. It is partly true that students will be able to do computation if they are drilled but will not be able to do problem solving and application questions properly because the latter demands both procedural and conceptual understanding. (2014, p. 2).

Coming to this point is of utmost importance as it is the focal point of the international tests such as TIMSS and PISA. Even though the teachers discussed and reviewed the video of the session for a few times, tracked the mathematical performance of the students, studied their strengths and weaknesses, they did not tackle the main goal behind teaching mathematics sufficiently. We acknowledge that what the teachers discussed after the lesson was conducted was important and some crucial issues were raised, however the point that students understood the mechanics of mathematics governing addition but they were not able to solve problems when dealing with a real-life situation was overlooked, the point underscored as a part of mathematical literacy in international assessment programs (PISA, 2013). For example, the students knew how to do 4+5 but when they were asked how they could increase the number of their walnuts the majority of them did not know what to do, as only a few students participated in the communication with the teacher and the rest were silent having a doubtful expression.

To handle such a teaching and learning gap, teachers need to develop problems pertaining to real-life situations and strategies which can help students to problem-solve and reach a higher level of learning. These strategies could be in the form of mathematical modeling processes through which problem solvers might go to solve a problem (Stacy, & Turner, 2015). Also, providing students with opportunities to use manipulatives in order to facilitate hands-on learning has the potential to deeply engage students in learning activities (Gaff, Lyons, & Watson, 2011; Namukasa, & Gadanidis, 2010). In addition, more collaborative professional development and frequent lesson study sessions might be beneficial to set the stage for more mathematical discourse and consequently find a way to grapple with the problem.

Conclusion

Observations from the teachers' lesson study practice show that lesson study framework could have the potential of being an alternative to traditional professional improvement programs in Iran. It provides teachers with opportunities to talk about the unforeseen events that might arise while teaching and helps them reconsider their knowledge and beliefs of the subject matter, pedagogy, learners and themselves. With that being said, however, it should be taken into consideration that while applying the lesson study framework, various avenues should be considered and most importantly, as Fernandez (2005) stresses, we should ask ourselves what kind of professional development opportunities this framework could provide teachers with, to what extent they can take advantage of them and what is "educative" about it (p. 268).

Moreover, the lesson study model has a lot to do with learning mindsets and school culture at which it is being practiced, a point that Stigler and Hiebert (1999) highlight as

the most important to consider. That is, this model will work effectively only if participants are open to learning from one another. Therefore, in a context in which teachers are used to working individually and or without receiving any criticism, it is important to practice and encourage collaborative educational activities along with implementing such a model. As this study shows, the participant teachers admit that they had difficulties with others' comments and they did not feel comfortable with peer critiquing, but as they maintained working with each other they learned how to handle these barriers.

The Japanese model of lesson study could face challenges in its application to Iranian educational contexts. Although lesson study has a certain framework and clear steps to take, it requires teachers to have a basic knowledge of pedagogy, subject matter and understanding of the context. Without them, teachers only carry out a cycle of steps leaving students with hollow promises. In order to lessen these difficulties, the principal should accept and implement a democratic and more relaxed management so that teachers could feel comfortable to express their ideas and practice what they think is appropriate.

The authors of the present study think more time is needed to examine the lesson study applicability in the context of Iran even though this study portrayed a promising picture. The teachers seemed interested in what they had learned in the course of lesson study and its results as professional development, however it is too early to claim that the lesson study model could be successful in Iran. We need to implement lesson study in public schools that have a radically different governance system. We also need to compare and contrast the Japanese culture of instructional designing with that of Iran to come up with more essential strategies that support reframing collaborative settings as the basis of lesson study to conduct in practice.

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