Width, length, and height conceptions of students with learning disabilities

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Teaching responsive to the needs of students with learning disabilities (LD) can be provided through understanding students' conceptions and their ways of learning. The current research, as a case study based on qualitative design, aimed to investigate the conceptions of students with learning disabilities with regard to the different representations of length (width, length, and height). The participants were 3 Turkish students at 4th, 5th and 6th grades recognised by the local Counselling and Research Centre (CRC) as having learning disabilities. The data were collected through semi-structured and task-based interviews, and analysed via content analysis method. The findings suggest that width, length, and height conceptions of students with learning disabilities are influenced by their conceptions of length and visual-spatial abilities.

Introduction

Learning disability is defined as showing unexpected and atypical learning failure without explicit causes (Fuchs, Mock, Morgan & Young, 2003). 6% or 7% of the student population (Berch & Mazzocco, 2007; Geary, 2011) and more than 50% of the students in special education consist of students with LD (Smith, Polloway, Patton, Dowdy & Doughty, 2015). Students with LD are more densely populated in inclusive classes in Turkey than students with other diverse disabilities (The Ministry of National Education of Turkey (MNE), 2010). The principle of equity in education requires providing students with LD qualified learning experiences with high expectations as for all students (Individuals with Disabilities Education Act (IDEA), 2004; Basic Law of National Education of Turkey (BLNE), 1973; National Council of Teachers of Mathematics (NCTM), 2000; No Child Left Behind Act, 2002). Another important principle is the needs of the individual and the society. It requires an education system responsive to the needs and differences of individual students (BLNE, 1973). However, one of the most complex and challenging issues to address and deal with in education is to respond to equality and diversity expectations (Cobb, Hodge & Gresalfi, 2011).

Responsive teaching requires deep understanding and knowledge about students (Jacobs & Empson, 2016). One of the actions to be taken in order to recognise students with LD and promote responsive teaching is to reveal their mathematical thinking, different perceptions and conceptions. Since the studies conducted for this purpose focused mostly on basic arithmetical and numerical competencies, the literature lacks investigation of what kind of thinking and conceptions students have for different mathematical concepts (Lewis, 2014; Rousselle & Noël, 2007; Woodward & Montague, 2002). Investigating more complex mathematical issues that cannot be evaluated with rapidity and accuracy is required for better recognition of LD and presenting solution proposals (Lewis, 2014).

Moreover, Cawley, Foley and Hayes (2009) emphasised that the studies on students with LD need to focus on measurement and geometry. These learning domains are significant and functional for daily life, and offer opportunities to improve cognitive performance, communication, and language comprehension (Cayley, Foley & Hayes, 2009). Measurement, which is the keystone of geometry (Zacharos, 2006), is also important for quantitative reasoning involving ratio, proportions and correlations, as well as for daily life skills (Sarama, Clements, Barrett, Van Dine & McDonel, 2011). Accordingly, this study aimed to investigate the conceptions of students with LD with regards to width, length, and height concepts which are different representations of length.

Learning disabilities

It appears that it is difficult to define LD simply or fully and to explain its reasons, and there is no consensus on this issue (Büttner & Hasselhorn, 2011; Lewis, 2014; Mazzocco, 2007; Zeleke, 2004). According to the MNE, LD is defined as "difficulty that a child has in acquiring and using literacy, mathematical-arithmetical skills, speaking-listening and reasoning skills" (2014). According to the United States,

> "specific learning disability" means a disorder in one or more of the basic psychological processes involved in understanding or in using language, spoken or written, which disorder may manifest itself in an imperfect ability to listen, think, speak, read, write, spell, or to do mathematical calculations (IDEA, 2004).

Learning disability can be defined as showing unexpected and atypical learning failure without explicit causes (Fuchs et al., 2003). Other criteria for determining LD are the existence of no evidences for the cause of difficulty in learning (such as education, environment, IQ, language) (Fletcher, Lyon, Fuchs & Barnes, 2006) and failure in the standard education provided to peers and appropriate for a student's age (Fletcher et al., 2006; Fuchs, Fuchs, & Compton, 2012).

Studies on LD generally focus on defining LD, revealing the underlying reasons, and conceptualising the student insufficiency (e.g. Butterworth & Laurillard, 2010; Geary, 2004; 2011; Geary, Hoard, Byrd-Craven, Nugent & Numtee, 2007; Geary, Hoard, Nugent & Byrd-Craven, 2008; Lewis, 2014; Mazzocco, Feigenson & Halberda, 2011; Piazza et al., 2010; Toll, van der Ven, Kroesbergen & van Luit, 2011; van Garderen, Scheuermann & Poch, 2014). According to Lubienski and Bowen (2000), studies conducted between 1982-1998 were carried out in psychology rather than mathematics education, and more than half of them dealt with cognitive processes, and they were not related to the class environment and affective domain of students. Similar studies exist in the related literature for the following years (e.g. Compton, Fuchs, Fuchs, Lambert & Hamlett, 2012). These studies generally compare LD students through large-scale tests with their low achieving peers who have not been diagnosed as having LD (e.g. Geary, Hoard, Nugent & Bailey, 2012; Rousselle & Noël, 2007; Toll et al., 2011). Considering that large-scale studies in the literature are not sensitive to individual differences, qualitative studies enabling a deeper investigation into the characteristics and differences of students with LD become important (e.g. Heyd-Metzuyanim, 2013; Lewis, 2014).

Width, length, and height

Length is the distance between the endpoints of a linear object; or when a nonlinear object is made linear, the distance between the endpoints is the length of the object (Argün, Arıkan, Bulut & Halıcıoğlu, 2014). Accordingly, length is a measurable attribute of objects. Therefore, length can be considered as a context in which the concept of measurement is observed. In this regard, the construction of the length concept must take place within the framework of the characteristics of measurement concept. The characteristics of measurement are recognition of attribute, the principle of conservation, transitivity, appropriateness of unit, equal units, unit iteration, and the relation between number and measurement.

A student should first recognise the attribute of length to comprehend it. The student needs to recognise different representations of the attribute such as height, perimeter, and distance (Outhred, Mitchelmore, McPhail & Gould, 2003). These representations of length include width, length, and height. The concepts of width, length, and height are respectively defined as "the distance between the two sides which are regarded as the length on a surface", "the distance between the two sides which are regarded as width on a surface" and "the distance between the base and the vertex of geometrical objects" (adapted from Turkish Language Institution (TLI) definitions). Width, length, and height also refer to each dimension of a 3-dimensional object.

The concepts of width, length, and height as different representations of length are not addressed in the teaching program developed by the MNE (MNE, 2013b). Similarly, there is limited research literature which focuses on these concepts as different representations of length, and directly examine conceptions regarding these concepts. Students' understanding of width, length, and height has been examined during the investigation of dimension and understanding volume and area formulas (Ebersbach, 2009; Paksu, Musan, İymen & Pakmak, 2012; Ural, 2011). More specifically, height has been handled as a geometric concept (Gutiérrez & Jaime, 1999; Gürefe & Gültekin, 2016; Hershkowitz, 1987).

In Euclidean geometry, the notion of dimension is determined by investigating which one(s) of the width, length, and height the object has (Skordoulis, Vitsas, Dafermos & Koleza, 2009). Accordingly, students should recognise and comprehend width, length, and height in order to understand dimension (Ural, 2011). Understanding dimension is essential for students to understand length, area and volume. To understand the properties of the attributes of length, area and volume such as covering space and spatiality, and the relation between them, it should be considered that length is 1-dimensional, area is 2-dimensional, and volume is 3-dimensional. This awareness is important to develop an idea of length, area and volume, and to differentiate between these attributes (Kamii & Clark, 1997; Lehrer, Jenkins & Osana, 1998). Similarly, the comprehension of length, area, and volume supports the comprehension of dimension. In the study by Ural (2011) investigating the criterion used by prospective mathematics teachers to determine the dimension, the criterion for majority of prospective teachers

(39%) was which one(s) of width, length, and height the object has, and the criterion for 25% of prospective teachers is whether the object has area and volume. Additionally, conceptions of width, length and height are important for making sense of area and volume formulas. Ebersbach (2009) has showed that children including those at a kindergarten (students with 3-6 ages) realise and consider width, length, and height in volume calculation. Accordingly, width, length and height, which are different representations of length, are important concepts in terms of reinforcing the student intuitions about dimension as well as giving meaning to area and volume formulas (Ebersbach, 2009; Skordoulis et al., 2009; Ural, 2011) and, hence, contributing to conceptually differentiate between the attributes of length, area and volume (Kamii & Clark, 1997; Lehrer, Jenkins, & Osana, 1998).

Rationale and purpose of the study

The in-depth examination of the conceptions by students with LD and the conceptions underlying their difficulties, misconceptions, and errors is crucial for better recognising students and providing a way of education that responds to their needs. In terms of cognitive contributions of the measurement and the difference of measurement conceptions from those of numerical skills and basic arithmetical calculations, this study aims to investigate the conceptions of students with LD regarding width, length, and height, which are different representations of length.

Method

This research is a case study based on qualitative design (Stake, 1995; Yin, 2013), investigating the width, length, and height conceptions of students with LD. It portrays the existing conceptions of students with LD without any intervention (Stake, 1995; Yin, 2013). Students with LD constitute the case of the research. The conceptions of students concerning width, length, and height are the units of analysis.

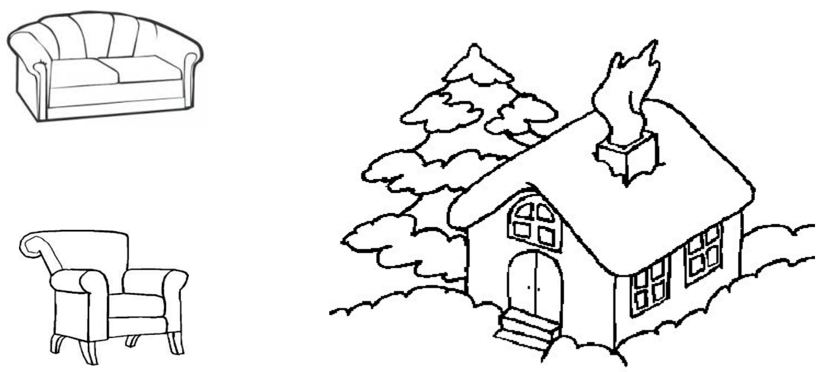
Participants

Participants were determined by criterion sampling and convenience sampling which are among the purposeful sampling strategies. The first criterion was that students were recognised by the CRC to have LD. In addition, in accordance with the information obtained from the teachers of students, the important point was to select participants who had reading skills at an instructional level (reading accurately without spelling). The participants of the study were three Turkish students with LD at the 4th, 5th and 6th grades. In the Turkish education system, children at the age of 5-6 start their formal education. Thus, the participants were respectively 9, 10, and 11 years-old. Emin was a 4th grade male, Merve was a 5th grade female and Fatih was a 6th grade male student (the names are pseudonyms).

Data collection

The research data were collected through semi-structured, task-based interviews, in which researcher and participants interact within a task (Goldin, 2000), and through the video and voice recordings of these interviews. The field notes were taken by the researchers during and after the interviews. The field notes and worksheets of students constitute the document data. Interviews were conducted in 2-3 sessions lasting for 40 minutes on average with each student individually. A pilot study was carried out with a student with LD to ensure that interview questions and tasks were comprehensible and meaningful for the students and appropriate for the purpose of the study. After the pilot study, some interview questions and tasks were changed, and new questions and statements were added where necessary. Firstly, the students were asked to explain width, length, and height and to show them on the given objects. Then, the students were asked whether verbal examples like "height of aquarium" and "width of table" were length or not. Measurement starts with comparing real objects, and measurement schemas of students become more extensive through factual situations based on comparison (Barrett & Clements, 2003). Therefore, in the next step, two tasks called "Robinson Crusoe and his adventures" and "Does it fit in or not?" that demanded making direct or indirect comparisons were performed. In the tasks, students were provided with a piece of paper, a pencil and a ruler, and they were asked to explain their decisions on whether given objects fit into the space or not. Whilst the Robinson Crusoe task was carried out with concrete materials, the other task was performed on the 2-dimensional representations of 3-dimensional objects. An example of the problems involved in the tasks is as follows.

> Examine whether the objects in the picture fit into the door of the house or not. Explain your answer. What should be done to make the objects fit into the door of the house? What are various solutions? Explain what you consider.



Both tasks obliged students to identify width, length, and height, and to make direct or indirect comparisons. Therefore, tasks allow for examining how students identify width, length, and height and how they use these lengths in direct or indirect comparisons. Students need to identify at least two different dimensions to determine whether the object could fit into the given space or not. The students were asked to inquire various possibilities in the case that the objects did not fit into the space. For example, there were discussions on what different positions could be or what the lengths of the object should be to fit into. This helped students take the different lengths into consideration. Participants were aware of the fact that length would be conserved under the translation, rotation, and reflection transformations. Therefore, the tasks were meaningful for the students. The tasks examined whether the students could correctly identify the representations of length, whether they compared the same lengths in indirect comparison and which one(s) of the lengths were considered by the students.

Data analysis

The content analysis method was used to reveal the categories of students' width, length and height conceptions. Data were transcribed verbatim to be re-read and analysed. The patterns characterising students' conceptions were searched for in the students' utterances, representations and drawings, to create categories. The transcriptions, field notes, and the documents of tasks were analysed for identifying repeated patterns. Field notes and worksheets helped to illustrate and figure out students' behaviours. Thus, trustworthiness of the study is established through data triangulation (Patton, 2005).

Expert opinion for data analysis was received from an expert who was a PhD candidate on mathematics education. Thus, peer review as an external control mechanism (Lincoln & Guba, 1985) was practised. The agreement-correlation coefficients between the researchers and the expert were calculated to be 0.93. As an example for the revision in light of the expert opinions, Merve's drawings for width (see Figure 2(a) and 2(c)) were interpreted as showing that Merve tried to consider the longest distance instead of the diagonal.

Findings and interpretations

Students' conceptions of width

The students stated that width was a length. However, when Merve was asked to show the width of a prism, she pointed at the face of the prism as illustrated in Figure 1(a). She showed the related face of the prism, not a single edge or distance between the edges. When the researcher asked what she meant exactly, it was understood that she meant the "whole" face that she showed. However, she pointed only at edges while identifying the height and length of the prism (Figure 1(b) and 1(c)).

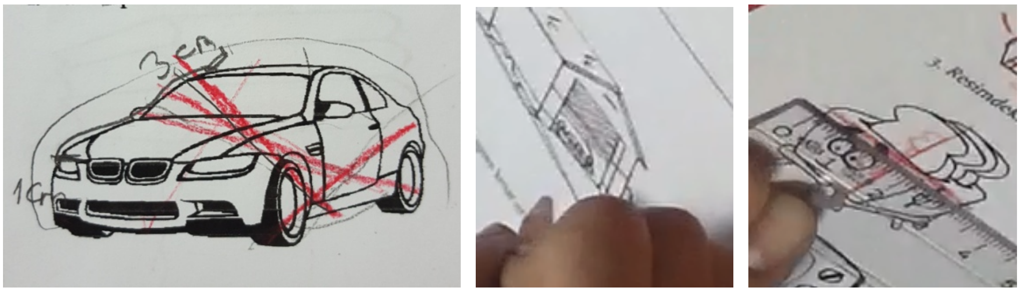
(a) (b) (c)

Figure 1: Merve shows width, length, and height respectively

Hence, it can be thought that although she searched for it in the correct orientation, she perceived width as a surface; however, she identified the width of all the visuals in the tasks by drawing a line (e.g. Figure 2(a), 2(b)). Accordingly, these different answers from Merve could be due to her lack of understanding the characteristic of length, that is, covering space in one dimension. Emin showed all the edges that can represent width and length except height for the width of prism.

In identifying width on the 2-dimensional representation of 3-dimensional objects, Merve and Emin made similar drawings as in Figure 2(a) instead of drawing parallel and perpendicular line segments to the edges representing the distance between the edges. The dialogue between the researcher (R) and Merve (M) regarding Figure 2(a) is given below.

R: Why did you draw it cross?
M: Well, if we consider it vertical like a wardrobe, this will be its width.
R: From where to where?
M: From here [showing the top of the car] to this wheel.



(a) (b) (c)

Figure 2: Width demonstration by Merve

While specifying the width, Merve might have intuitively thought that she should identify the widest length. As it is clear in her expression "If we think it vertical like a wardrobe", Merve tried to decide by comparing it with a rectangular prism. However, it is seen that the length she drew was closer to height. While measuring the garage in which the car can fit into, Merve identified the width of the garage easily (Figure 2(b)). The reason might be the garage is more similar to geometric objects she knows rather than the car, and the edges of the garage are more apparent. Nevertheless, she answered the question of which lengths she measured as "Here is width. Here is length [referring the length attribute]".

Drawings and measurement that participants performed to identify width on the 2-dimensional representation of 3-dimensional objects are exemplified in Figure 3. As seen, students identified width as the distance between the endpoints of the object horizontally. Students' width drawings are similar to a diagonal. In other words, students considered width and length as one length in these drawings to specify the width in the 2-dimensional pictorial representation of 3-dimensional objects. It can be figured out that students could not visualise the object or its edges when it is depicted as a picture, or they did not distinguish the edges of the objects. Students might have difficulty in imagining the 2-

dimension representation of 3-dimensional objects. For example, Emin said "I cannot do this […] because I am not there" once "Does it fit into or not?" task was presented to him. Emin, a 4th grader, was aware that a different dimension was practised with the 2-dimension representations of 3-dimension objects.

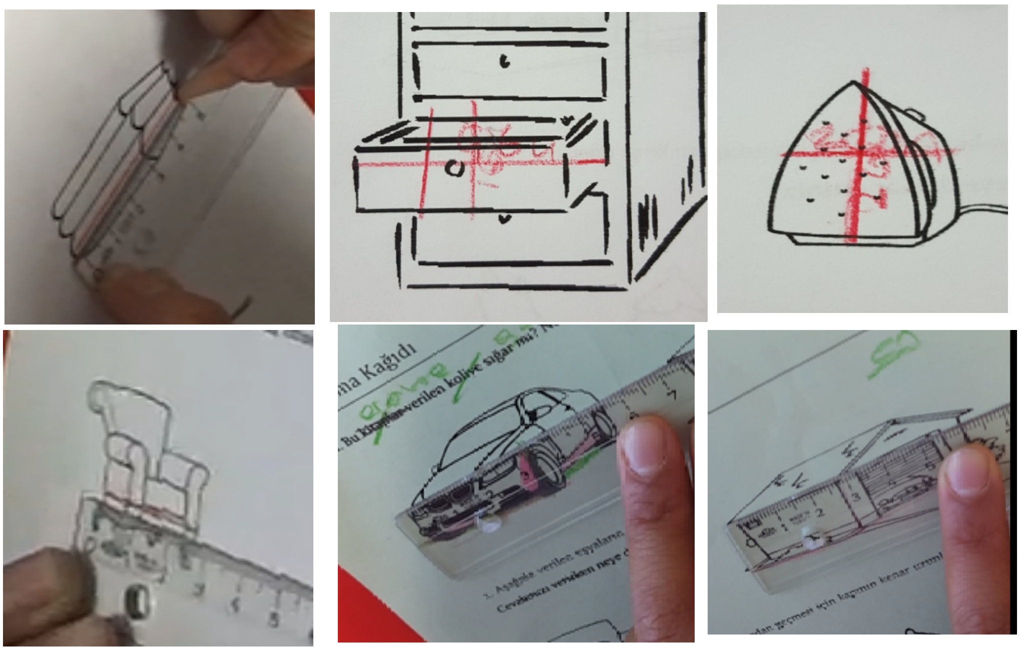


Figure 3: Width demonstration and measurements of the students

Students' conceptions of length

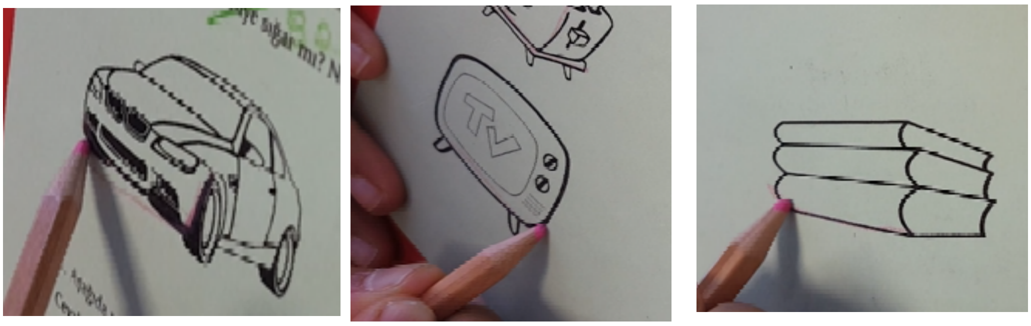
Whereas, in English, the term "length" is used to express one of the representations of length; in Turkish, this representation is verbalised with a different word, "boy", instead of the Turkish word for length. The "boy" which is one of the representations of length will be expressed as *length* from now on so that the findings would be more comprehensible and *length* can be distinguished from the length attribute. The students indicated that *length* is a length. When the researcher asked what length was, they told it was *length.* Indeed, they consider length as the *length* of an object; in other words, they oversimplify the concept of length to the representation of *length.*

Although students focused on *length* representation of length, but not the other ones, they had more problems in identifying *length* on objects than width and height. Emin matched the *length* with an object. When he was asked to show the *length* of a rectangular prism.

R: Can you show me its length?
E: Itself.
R: How?
E: It is *length*. Human *length*.

Emin indicated that the *length* of an object was itself. This idea of Emin might be related to the use of *length* in daily life. Similarly, he refers to its use in daily language.

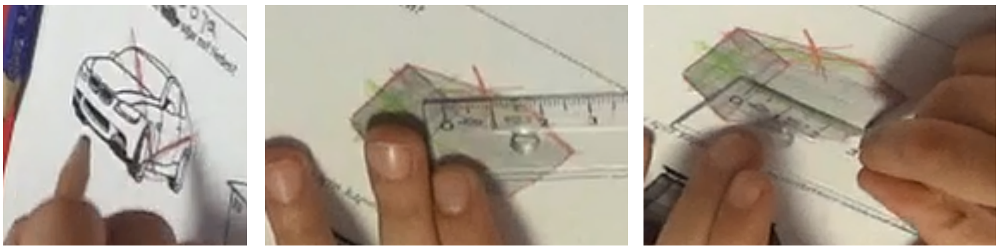
Merve showed the height (Figure 1(b)) or all the edges for the *length* of the rectangular prism. In most of their comparisons, Merve and Emin handled width and *length* together as mentioned under the title of width, and they did not measure *length* in the rest. When they were especially asked to elaborate on, it was observed that students confused *length* with height. It might also result from the use of the word *length* in daily language.



(a) (b) (c)

Figure 4: Length demonstration by Fatih

Another issue was that students tried to identify *length* by always looking at the same vertex, either left or right. As seen in Figure 4(a), 4(b), and 4(c), Fatih searched for *length* at the points that the pencil touched. This situation could be due to the students' over simplification in the way that they always need to look for *length* in a vertex.



(a) (b) (c)

Figure 5: *Length* demonstrations by students

Students did not specify the *length* as parallel to the edges as they do for width (Figure 5(a), 5(b), and 5(c)). Upon the measurement as seen in Figure 5(b), Emin was asked "Why are you holding the ruler like that?" Therewith, he changed its position but he held the ruler as in Figure 5(c), and thus, he could not make an accurate measurement in this way.

Emin had difficulty in measuring *length* - the depth of the bookcase. He tried to measure the depth of the shelf by holding the ruler as seen in Figure 6. He kept changing his way of holding the ruler because he thought the reason why he couldn't measure was the way of holding the ruler; however, it actually stems from the fact that he could not identify which length to measure.

Figure 6: *Length* measurement by Emin

Students' conceptions of height

Students stated that height was the length. Yet, Emin was not aware that height is conceptually the distance between the bases. When he was asked to show the height of the book in a horizontal position on the table:

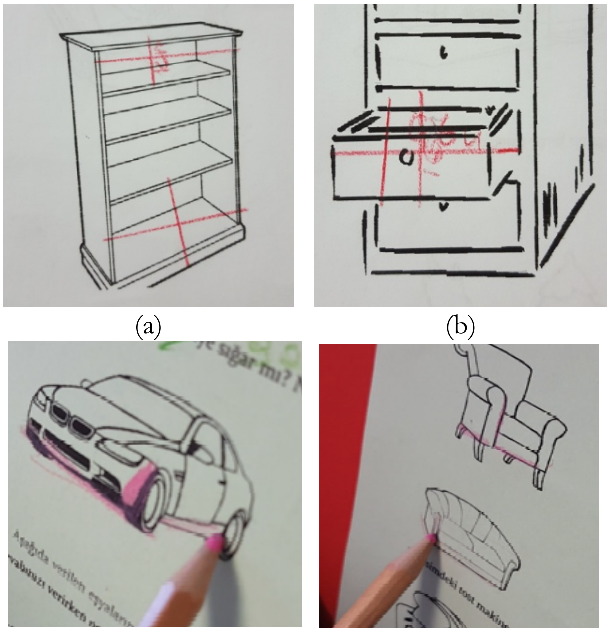
E: There is not height in this.
R: Why?
E: ...
R: How does it have height?
E: Here [he places the book in a vertical position (Figure 7(a))]
R: Then, what has the height?
E: Here we have [he shows the part of the book seen in the figure again]
R: Can you give another example for height?
E: This [he straightens the pencil sharpener which was placed horizontally on the table (Figure 7(b))]

(a) (b)

Figure 7: Height demonstration by Emin

As seen, Emin considers height as the situation of being high. As he was not aware that height is the distance between bases, he thought that height is looked for when there is a matter of being high. This can be understood from his statements that the book and the sharpener had height when they were made vertical. Emin associated height with being high, but he was not aware that it is relative to the positions of objects. Although Emin expressed height was a length, he showed only the higher point instead of showing the distance. This may suggest that his conception of length was not sufficient. When Merve was given a prism block and asked about the height, she said she did not know where the height was. When she was asked: “Suppose it is a refrigerator. Can you show its height?” She showed the two opposite edges together as seen in Figure 1(c). During the interviews, students usually expressed that the length they identified for height was the *length*.

The students had similar conceptions of height as those of width and *length*. For example, Figure 8(a) exemplifies that they did not draw line segments which were parallel to the edges forming the height, or perpendicular to the bases; Figure 8(b) exemplifies that they handled height and *length* together, or they drew things similar to the diagonal; 8(c) and 8(d) exemplify that they ignored the longest distance while determining height.



(a) (b) (c) (d)

Figure 8: Height demonstrations by students

As a summary of findings up to this point, students with LD may not be aware of covering space in the one-dimension characteristic of length. They over simplified the length to its representation-*length*. Despite this, they had more trouble with identifying *length* of an object than width and height. They were more successful in recognising lengths on 3-dimensional objects than 2-dimensional representations of them. They had difficulty in distinguishing the edges that can be seen in 2-dimensional representations of 3-dimensional objects.

Width, length, and height in indirect comparison

This part examines whether the students compared the same lengths while making indirect comparisons, and which one(s) of the lengths they addressed. As stated before, firstly the *Robinson Crusoe and his adventures task* was presented to the students. In this task, the objects that Robinson should decide on whether they fit into or not were represented by using objects in the classroom where the interviews were conducted. For example, the cave entrance found by Robinson was represented by the door of the classroom, and the things Robinson wanted to pass through the entrance of the cave were represented by the table and the bookshelf in the classroom. Emin decided to check whether the bookshelf passed through the door or not:

E: [...] Then, he first measures the cave. Next, he goes and measures that, his belongings. Then he finds out whether it can pass or not [Emin mentions what Robinson can do].
R: What should he measure in the cave?
E: The entrance.
R: Where is the entrance?
E: Door.
R: For example, how do you measure the door?
E: The sides when it is open [opens his arms].

Emin stated that width of the door should be measured. He was asked to measure and compare the lengths he stated. He stood up and measured the width of the door. Then, he said "Let's measure the bookshelf", and he measured the height as seen in Figure 9.

Figure 9: Emin measures height of the bookshelf

As seen, Emin aimed at comparing the width of the door and height of the bookshelf. Merve showed a similar approach by comparing the *length* of the car and width of the door. However, students compared two different lengths (width and *length*/ width and height) once or twice during tasks. Therefore, it does not seem systematic, and the reason might be students' inattentive acts.

The students made a decision by considering only two lengths in their comparisons. It was observed that they made decisions based on width and height by ignoring *length* in the

“Does it fit into or not?” task which was applied by using 2-dimensional representations of 3-dimension. One of the reasons why students ignore *length* might be that students handle width or height together with *length* as explained above. Therefore, students decided only by focusing on two dimensions. It might stem from their perceptions of the pictorial representations of 3-dimension as 2-dimension.

Since students were not aware of dimension, they were not consistent regarding the number of lengths they specified. They did not hesitate to state 4 dimensional lengths that they thought as different. For example, when Fatih was asked to tell which lengths he measured while measuring the lengths of the car:

R: Which length are you showing?
F: Its length.
R: Width, *length*, height? Which one?
F: Its length [...] Width is here [showing width]. *Length* is here [showing *length*]. Height is here [showing height]. Here is length [showing the second line in the base again- *length*]

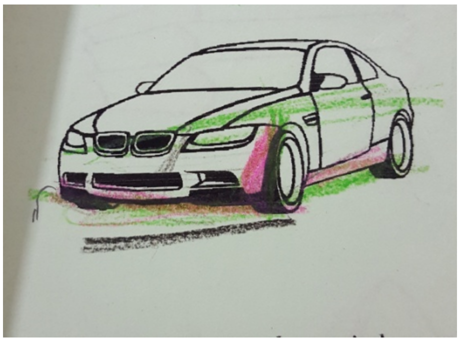


Figure 10: Lengths demonstration by Fatih

Looking at Figure 10, it can be seen that the *length* of the car was drawn twice. Fatih showed the line above while talking about "*length*" and showed the bottom line while indicating “length”. Fatih felt the need to rephrase *length* as length again. When he was asked whether the other ones were length or not, he said they were also the length.

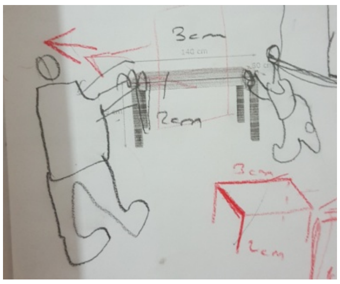


Figure 11(a) The representation by Emin for carrying the table

Figure 11(b) The representation by Merve for the couch with an eraser

When the students were asked under which conditions the objects could fit in, they could determine how the measurement of lengths should be changed; however, they were unable to make decisions by imagining in spite of receiving help from the concrete objects. For example, the way of passing the couch through the door was exemplified with the help of an eraser. Merve decided correctly on how the couch could be passed with the eraser. However, while applying it to the picture of the couch, she measured width and height as in the first measurement instead of height and *length* that she should have measured. The same problem was observed in other students' measurement. For example, height-height comparison should transform into height-*length* in the new position; however, students compared height and height again. Therefore, it can be alleged that students use mental figures in the measurement activities (Sarama et al., 2011), but they cannot imagine the change while comparing the lengths of the objects when they are replaced in a new position.

To summarise the findings under this title, students with LD generally compared the same lengths while making indirect comparisons. However, they made decisions by only comparing 2 dimensions. They sometimes unwittingly may count 4 dimensions for an object. They had difficulty in mentally transforming objects, especially in the 2-dimensional representations of 3-dimensional objects. Thus, this affected their performance on the comparison of lengths.

Results and discussion

Width, *length*, and height concepts are the representations of the length attribute, and the recognition characteristic of the attribute includes recognising various representations of it. The recognition of attribute is targeted from preschool to 3rd or 4th grades (MNE, 2013a, 2013b; NCTM, 2006; Outhred et al., 2003). The findings of this study suggest that the width, *length*, and height conceptions of students with LD and identifying width, *length*, and height of the objects are influenced by students' conceptions of length and their visual-spatial abilities.

Although students expressed that width, *length*, and height are a length, the reason underlying the statements like "Here is width. Here is length" is thought to be students' recognition of length as *length*. Students have the perception that "length equals *length*". This perception is undeniable that the most common length representation students encounter in daily life is *length,* and the concept of *length* is used more commonly than the other representations of length. Moreover, the longest dimension of a 3-dimensional object is stated as its length. Therefore, the expression of length is used to express one of the representations of length attribute. This representation is thought to be *length* generally in Turkish like in English (e.g. human *length*). The definition of width as "opposite of length, *length*" by TLI (2011) is an example of the interchangeable use of *length* and length. Similarly, the reason why students express height measurements as *length* might be due to daily language. Hence, words have an important effect on learning length (Sarama et al., 2011). Under these circumstances, there might be a possibility of a cognitive conflict like "Width is a length. Length is *length*. Then, width is *length* but I am showing different lengths" for students. This conception might restrict students' conceptions of other representations of length. This can explain why students became more confused in determining the *length* of an object than in determining width and height.

Students generally did not consider the widest-longest distance while determining the lengths of objects of which opposite edges are not equal, such as car and couch. It might stem from the fact that they do not conceptually address height as a dimensional length. Besides, the reason why Emin's search for the height in the case of "being high" might be a perception related to the word stem in Turkish (the stem of height is high in Turkish) and his unawareness about height as a distance.

Students were more successful in determining and comparing the lengths of 3-dimensional objects than 2-dimensional representations of 3-dimensional objects. There might be three reasons why the students drew lines not parallel to the edges representing width, height, and *length* instead of drawing the distance between edges while showing width, height, and *length* on 2-dimensional representations of 3-dimensional objects. The first one could be inattentiveness to this issue; the second one could be they perceive that the length they drew is the exact length that represents the distance; the third one could be they were unaware that the line they drew was not equal to the edges that represented lengths. Therefore, as expected, conceptions of length influence the conceptions of width, height, and *length*.

While identifying the lengths, drawing lines not parallel to the edges representing these lengths is different from drawing the similar lengths to the diagonal. The student who draws lines not parallel to edges searches for the lengths in the correct orientation but does not draw a line parallel to the edge or keep the ruler straight in the measurement. It is fundamentally thought to be a result of the conception of length as explained above. Drawing a length similar to the diagonal might result from students' ignoring the edges of objects or the fact that they cannot distinguish the edges on the planar representations of 3-dimensional objects. Therefore, students might have problems in differentiating the edges that are constituents of 2-dimensional representations of 3- dimensional objects. Likewise, students without LD have difficulty in identifying edges more generally in reasoning over 2-dimensional representations of 3-dimensional objects (Emül, 2013; Pittalis & Christou, 2010; Ryu, Chong & Song, 2007). However, while the related literature indicates that students have difficulty in identifying the edges that are not seen, the participants of the current study have difficulty in distinguishing the edges that can be seen. Therefore, students evaluate the objects as 2-dimensional by handling width and *length* or height and *length* together in the 2-dimensional representations of 3-dimensional objects. It can be asserted that students are not aware that dimension is represented with dimensional lengths. The reason for that could be their insufficiency regarding the conceptions of dimension.

Focusing on only two dimensions in the indirect comparisons or stating 4 lengths is thought to be related to students' spatial reasoning. Therefore, students search for the *length* of the object always at the right or left corner. While identifying the width, they focus on considering the edge of the base every time. Conversely, Owens (2004) stated that 4th and 6th grade students without LD can employ reasoning about 2-dimensional representations of 3-dimensional objects, however, attention and focus influence this reasoning (Owens, 2004). The participant students do not have the diagnosis of attention deficit disorder with hyperactivity (ADDH); however, students with LD have problems in paying attention to something and in keeping doing it even if they do not have ADDH (Sterr, 2004). Therefore, non-systematic errors of participants in reasoning about 2-dimensional representations of 3-dimensional objects might be due to their attention problems.

When students decide that objects did not fit into the given space while examining 2-dimensional representations of 3-dimensional objects, they decide on rotation by using concrete materials. They can identify the proper position with concrete objects but have difficulty in imagining the rotation of the picture representation of a 3-dimensional object. According to Clements and Sarama (2014), one of the most important transformations that students should learn is mental rotation. However, mentally transforming or manipulating objects is a part of visual-spatial abilities and requires spatial working memory (Lawton, 2010; Zhang, Ding, Stegall & Mo, 2012). It is challenging for students to decide on which length they should measure in a new mental position, and then to remember and identify it when returning to the original position on paper. The difficulties of students might be related to their spatial working memory. It is stated that visual-spatial deficits might be one of the characteristics of learning disability (Geary, 2004; Rousselle & Noël, 2007). Although the learning disabilities of participant students is not based on the deficits in visual-spatial working memory, visual-spatial abilities of students with LD may be lower than their peers (Andersson, 2010; Grobecker & De Lisi, 2000; Mammarella, Giofrè, Ferrara & Cornoldi, 2013).

Conclusions and implications for mathematics education and further studies

While indicating the importance of having high expectations in measurement and geometry for students with LD (Cawley et al., 2009), the fact that there are a limited number of studies on students with LD in these domains draws attention (Cawley et al., 2009; Lewis, 2014; Woodward & Montague, 2002). The current study is an illustrative study into the conceptions and thinking of the students with learning disabilities regarding various concepts of measurement through direct or indirect comparisons that are non-numeric strategies of measurement (Sarama, Clements, Barrett, Van Dine & McDonel, 2011). It is significant for recognising students with LD and revealing their difficulties in different learning domains.

The notion of length is one of the fundamental attributes of measurement that offer opportunities for improving language, communication, and cognitive performance, which are the essential components of daily life. This study contributes to the literature in providing an insight about the conceptions of students with LD in terms of different representations of length. Moreover, the findings of the study can give insights for the visual-spatial abilities of students with LD in differentiating edges and mental rotation of the representation of 3-dimensional objects on the plane. However, there is a need to investigate visual-spatial abilities of students with LD in detail. The number of studies thoroughly examining the visual-spatial abilities of students with LD as a mathematical skill (e.g. Andersson, 2010; Grobecker & De Lisi, 2000) is less than those examining this ability by associating it with spatial representation of numeric values or numeric data processing (e.g. Geary, 2004; Geary et al., 2007; Geary et al., 2008). There is a need for studies on LD from the perspectives of mathematics education (Lewis, 2014; Lubienski & Bowen, 2000).

According to the findings, as implications for mathematics education, students should be convinced that width, *length*, and height are different representations of length, and *length* is one of the representations of length by referring to the definition of length. Investigating objects not only with equal edges but also with unequal edges is thought to be important for understanding width, *length*, and height. Also, it is thought to be important to make students realise that 3-dimensional objects only have width, height, and *length* for intuitive understanding of dimension.

Although the "technical lexis" characteristic of length, which means using the word of length in the same meaning in daily life, is regarded more advantageous than area and volume (Zacharos, 2006, p. 225), conceptions of length might be negatively influenced by daily language as seen in *length*. Additionally, given that one of the students associated height directly with being higher, paying attention to the words used in teaching can make it possible to predict the difficulties of students with LD that might originate from language and to avoid them.

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