A framework to assist mathematics teachers in integrating problem solving in secondary school classrooms

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Successful integration of problem-solving in mathematics classrooms lies mostly with the assistance and support teachers get. Many studies have been conducted on the strategies teachers can use to integrate problem-solving in their teaching of mathematics, however, few studies have been conducted on frameworks/models that can assist teachers and students to integrate problem solving in mathematics. There is a need for the development of framework(s) that focus on helping teachers and students with problem-solving in classroom activities and practices. The study reported here was undertaken to explore teachers' viewpoints on the classroom practices that help develop learners' problem-solving skills in mathematics, with the purpose of developing a framework that can be used in the integration of problem-solving activities in mathematics classroom teaching. A qualitative case study that involved five secondary school teachers was undertaken in the Maseru district, Lesotho. Data were generated through an open-ended questionnaire, semi-structured interviews, and lesson observations and were analysed using thematic analysis. The findings revealed that the teachers possess relevant information about problem solving, but are challenged in integrating related practices and activities in their teaching. The amount of time available during mathematics lessons and the use of real-world problems were found to be among the most difficult issues. Most importantly, the findings provided useful information that helped in developing a framework that mathematics teachers could use to integrate problem-solving in their classroom activities.

Introduction

Problem-solving is a core aspect of mathematics curricula around the world (Bostic, Pape & Jacobbe, 2016). It is a goal and outcome of the learning and teaching process (Aljaberi & Gheith, 2016). This means that meaningful teaching and learning of mathematics in the classrooms takes place in the context of problem-solving where learners are helped and supported to explore mathematical facts, concepts, and skills they should be learning and to apply their gained mathematical knowledge and experiences in new and/or unfamiliar situations.

Mathematical problem-solving is a process through which learners explore an unfamiliar and non-routine task for which the solution method is not obvious and/or known in advance (Rott, 2012; National Council of Teachers of Mathematics, NCTM, 2000). Solving problems is the pronounced manifestation and usefulness of mathematics (Căprioară, 2015) and it helps learners to develop mathematical knowledge and also enhances the generic ability to solve real-world problems (Guzman Gurat, 2018; Hwang & Riccomini, 2016).

However, “problem-solving… does not seem to be a skill that is widely practiced and nurtured within classrooms” (Bradshaw & Hazell, 2017, p. 32). Few teachers have an idea of exactly what problem-solving is, and how it should be taught and assessed (Atteh, Andam & Obeng-Denteh, 2017). Some teachers are reluctant to integrate problem-solving activities into mathematics teaching, and those who would want to use problem-solving approaches are unsure how to go about it (McIntosh & Jarrett, 2000). Teachers are not usually comfortable with problem-solving activities (Govender, 2015).

If teachers continue being challenged to integrate problem-solving in the classrooms, then mathematics teaching in the classrooms will continue to be teacher-directed and content-focused, emphasising memorisation of facts, concepts, and mastery of algorithms. This type of teaching approach fails to consider learners as independent thinkers who can develop their knowledge, and therefore, does not help learners develop problem-solving skills. Lecturing is a popular and common teaching approach in the classrooms and this does not bring about the form of learning that encourages learners’ critical thinking and problem-solving (Atteh, Andam & Obeng-Denteh, 2017).

Despite the challenges the teachers seem to be facing, there is limited literature on the frameworks that may assist them in integrating problem-solving in their teaching of mathematics. This gap in the literature motivated and forms the basis for this focus in the study. We argue in this article that for the teachers to integrate problem-solving activities in their mathematics classrooms, the existence of a framework that guides them would go a long way towards improving this situation. Most importantly, the teachers’ viewpoints on the practices that can help develop learners’ problem-solving skills in mathematics should be taken into account and used as an important building block for the framework.

Aim and research questions

This study aimed to develop a possible framework that can be used to assist teachers in the integration of problem-solving activities in their classrooms, guided by the following questions.

Main research question
What framework can be developed to enhance the integration of problem-solving in mathematics classrooms?

Sub-questions
1. What are the teachers’ viewpoints on the classroom practices that help develop problem-solving skills in mathematics?
2. How can the viewpoints and classroom practices be used to develop a framework to assist learners in problem-solving in mathematics classrooms

Literature review

Importance of problem-solving in mathematics teaching and learning

"Problem-solving is the manifestation of constructivist learning in mathematics, which is the idea that students learn best through constructing their knowledge" (Evans, 2012, p.35). This means that through solving problems, learners get opportunities to relate and connect mathematical facts, concepts, skills, and ideas to develop a deeper mathematical understanding and its application. That is, in problem-solving learners should not practise memorised algorithms, but rather, develop their new understanding of mathematical facts and concepts, and therefore, in solving problems they learn to analyse the given information, argue and reason their mathematical knowledge and thinking. In solving a problem, learners employ a range of strategies to get a solution, and as a result, they develop processes of analysing, reasoning, generalising, and abstracting (Anderson & White, 2004). Problem-solving encourages learners to develop a deeper understanding of concepts (Guzman Gurat, 2018, p. 53) to relate and to use the mathematics that they have learned in everyday life situations (Guzman Gurat, 2018; Akinmola, 2014). Therefore, problem-solving enables learners to develop skills to tackle problems that arise in the real-life context (Guzman Gurat, 2018; Osman et al., 2018).

Pólya (1945) developed the most known and cited mathematical problem-solving model that describes the process and guides learners in solving unfamiliar mathematical problems (Evans, 2012). Later, Schoenfeld (1985) developed a slightly modified model. Figure 1 compares the two models.

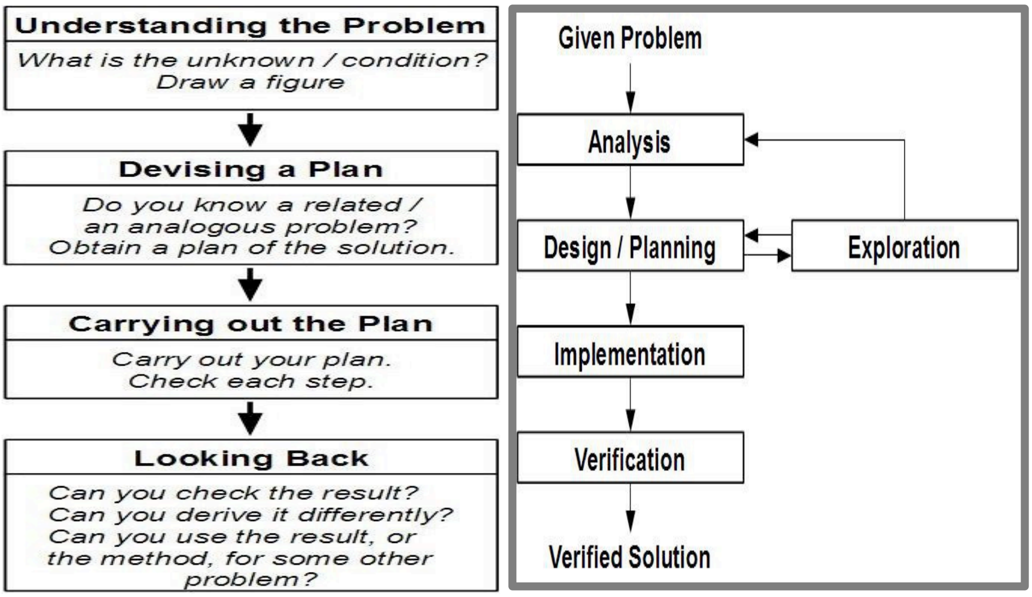


Figure 1: Pólya's (1945) model (left) and Schoenfeld's (1985) model (right)
(adapted from Rott, 2012, p. 97)

Atteh, Andam & Obeng-Denteh (2017) developed a framework, also similar to Polya's model, that can be implemented in any mathematics lesson to assist learners to develop their problem-solving skills. Their framework consists of 4-steps: (i) holistic understanding of the problem; (ii) identifying method(s) for the solution; (iii) applying the method(s) for a solution; and (iv) checking the accuracy of the solution. It was built based on existing ideas proposed over the years in the mathematics education discipline, the best practices concerning cognitive development, and an effective teaching and learning environment including solved examples, that provides teachers with very useful guidelines for classroom instruction.

Despite the differences in the number of steps/components each model consists of and the naming of such steps, all models have very similar ideas about what the learners need to do during problem-solving. In particular, learners need to (i) explore and understand the problem; (ii) come up with a solution strategy and implement it in solving the problem; and (iii) verify, reason, and justify the correctness of their solutions and the procedure used. That is, to solve the problem successfully, learners should be able to answer these three interrelated questions "what information is given in the question to help me find the solution?"; "how do I find the answer to the problem?"; and "why does my answer and solution procedure make sense?". The key similarity in the models suggests that problem-solving is a process and that the learners cannot simply apply it to a previously learned procedure and/or memorised algorithm to solve the authentic mathematical problem.

Teachers' challenges in integrating problem-solving in mathematics teaching

Despite the importance of mathematical problem-solving in teaching and learning, and in real-life, its teaching "is rarely present in the formal teaching environments" (Aljaberi & Gheith, 2016, p. 32). This means that teachers are still challenged in integrating problem-solving in mathematics classrooms. Doorman et al. (2007, p. 418) argued that teachers are mostly challenged by designing good problem-solving tasks that are authentic, non-routine, and new to the learners. In mathematics classrooms, the learning task enhances students' learning when it encourages and requires them to engage in high-level thinking and reasoning as opposed to those tasks that are merely routinely procedural (NCTM, 2014).

The nature of mathematical tasks that teachers give to learners in the classrooms allows students an opportunity to learn and determine the content that they learn (Stein, Grover & Henningsen (1996). That is if a learner can readily recognise the solution procedure then the task is simply a routine mathematical exercise. This means that real mathematical problems are the backbone of teaching problem-solving in the classrooms and learners' construction of mathematical knowledge, concepts, facts, and skills should be developed through solving problems.

> Teachers must decide on the problems and problem-solving experiences to use, when to give problem-solving particular attention, how much guidance to give students, and how to assess students' progress (DiMatteo & Lester 2010, p.7).

However, in problem-solving teaching "the teacher does not teach the detail so that students will find it difficult to find an understanding of the details" (Aljohani, 2017, p. 98), but rather, teachers should consider themselves as listeners, guides and observers, instead of an authority and solution giver (Norton, McRobbie & Cooper, 2002, p. 37). That is a teacher must be a facilitator and assist and support the learners in constructing their knowledge, rather than transmitting that knowledge.

In addition, learning, according to Vygotsky, is dependent on collaborative, discourse, and social interaction activities (Mohammed & Kinyo, 2020). Learners construct their knowledge individually and collectively whereby they and the teacher provide the context, pose questions, present challenges, and support that encourages the learners' mathematical knowledge development (Davis, Maher & Noddings, 1990).

The argument put forth emphasises the importance of real problems and classroom interactions forming a backbone for the successful integration of problem-solving in mathematics classrooms. That is, the teachers' ability to design and/or choose real mathematical problems forms a backbone for creating problem-solving experiences for the learners. Authentic problems provide learners with opportunities to explore and understand mathematical facts, concepts, procedures, processes, and skills.

Factors influencing teacher's problem-solving practices in maths classrooms

Integration of mathematical problem-solving in the classroom lies mostly with mathematics teachers and an establishment of good mathematical beliefs could lead teachers to positive and effective teaching practices (Zakaria & Maat, 2012). The classroom's problem-solving practices and activities are dependent on the beliefs teachers hold about problem-solving (Harisman, Kusumah & Kusnandi, 2019; Bishaw, 2010; Ernest, 1989).

The adoption of problem-solving practices in the classroom is also influenced by the teachers' knowledge of teaching problem-solving, and therefore, the teachers must possess mathematical problem-solving knowledge for teaching, to be able to integrate it meaningfully into their classroom teaching (Chapman, 2015, p. 24). Chapman further highlights that the teachers' knowledge about mathematical problem-solving includes their ability to solve the mathematics problem, their knowledge of how an individual becomes a good problem-solver, and, most importantly, their ability to help learners develop their problem-solving skills.

The classroom practices that mathematics teachers employ are further influenced by their experiences, and the most influential mathematics teachers' experiences are those that originate from their learning as students (Minarni, Retnawati & Nugraheni, 2018, p. 3). It is likely that some of the teachers experienced traditional classroom instructions as learners and have a poor experience with problem-solving, and this, as a result, can make them find it very challenging to integrate problem-solving activities in their classrooms (Matić, 2017). These emphasise the need and importance of assisting the teachers to help them successfully integrate problem-solving in their teaching of mathematics in the classrooms. That is, even those teachers with a poor problem-solving background and/or low attitudes toward problem-solving may be willing to teach problem-solving, but can succeed only if they are supported with frameworks that can guide them on how to do it.

Medium of instruction in mathematics classrooms

In Lesotho, Sesotho is the first (mother tongue) language, but English is the official language of instruction in all subjects except for the languages (Sesotho, English, and French). Although participants mostly used English in their teaching of mathematics, they sometimes code-switched between English and Sesotho. Uys (2010, p. 59) highlighted that teachers code switch mainly for three reasons: academic purposes (such as explaining and clarifying subject content); social reasons (maintaining their social relationships with their learners and being humorous; and for classroom management purposes (such as reprimanding learners). In this study teachers mostly code-switched when explaining the concepts and/or confirming learners' understanding. Therefore, the participants' use of these two languages during teaching and learning lessons reflects the excerpts in the findings.

Theoretical framework

The theoretical framework used to guide this research is the *theory of didactical situation* (TDS) that was developed by Brousseau (1977). TDS was considered relevant in this study because it seeks to offer a model to explore, in a scientific manner, the problems related to the teaching of mathematics and how they can be improved (Radford, 2018; Artigue, Haspekian & Corblin-Lenfant, 2014) and it was used in this study to interpret teachers' viewpoints about classroom practices in problem-solving. The theory of didactical situations propounds the modelling of knowledge, situations of teaching, and the teachers' and learners' roles in the classroom (Delacour, 2012) and is concerned primarily with understanding the circumstances and restraints of didactical systems that permit or inhibit learning, and how the operation of such systems can be enhanced (Artigue, Haspekian & Corblin-Lenfant, 2014).

Method

This qualitative multiple case study was conducted in the Maseru district, the capital city of Lesotho. Five Grade 10 mathematics teachers selected through purposive sampling participated in the study. Purposive sampling allowed for the identification of information-rich participants who were willing to share information (Creswell, 2013). The teachers who participated in this study were from different schools and are qualified secondary mathematics teachers with more than five years of teaching experience and willingly volunteered to take part in the study. The data were generated at the teachers' respective schools from February to May 2021. Ethical clearance was obtained from the University of the Free State. Permission to conduct the study was also obtained from the Ministry of Education in Lesotho. The teachers and students who participated in this study signed consent forms that indicated that the data collected will be kept confidential and will be used for research and no other purpose.

Data collection

The data were generated through open-ended questionnaires, face-to-face semi-structured interviews, and lesson observations. The questionnaire was administered to each of the participating teachers to gain primary insights into their viewpoints on the classroom practices that help develop learners' problem-solving skills in mathematics classrooms. The interviews with each teacher lasted between 45-60 minutes and each participant was observed during the presentation of a lesson on a mathematics topic of focus at the time of the research. The interviews and lesson observations were audio-recorded. The questionnaire and interview data collection protocols were developed by the researchers based on the reviewed literature and the data were analysed thematically (Alhojailan, 2012).

Data analysis

Thematic analysis was used to analyse the teachers' viewpoints, meanings, and experiences in mathematical problem-solving. The purpose of thematic analysis is to identify, classify, analyse and report patterns (themes) within the data set (Braun, Clark & Weather, 2016; Braun & Clark, 2006) and to interpret their meaning and importance (Braun, Clark & Weather, 2016). In conducting the thematic analysis the researcher (i) familiarised with the data; (ii) generated initial codes; (iii) searched for themes; (iv) reviewed themes; (v) defined and named themes; and (vi) produced a report (Braun & Clark, 2006).

Quality criteria

Trustworthiness was established in this study through triangulation (Stenfors, Kajamaa & Bannett, 2020) and use of thick description and concrete detail (Tracy & Hinrichs, 2017) use of handwritten field notes (Cypress, 2017; Shenton, 2004), and providing rich quotes from the participants that depict each emerging theme (Cope, 2014).

Study findings

This section presents the study results generated by the teachers through the open-ended questionnaire, semi-structured interviews, and lesson observations.

Findings from the questionnaire

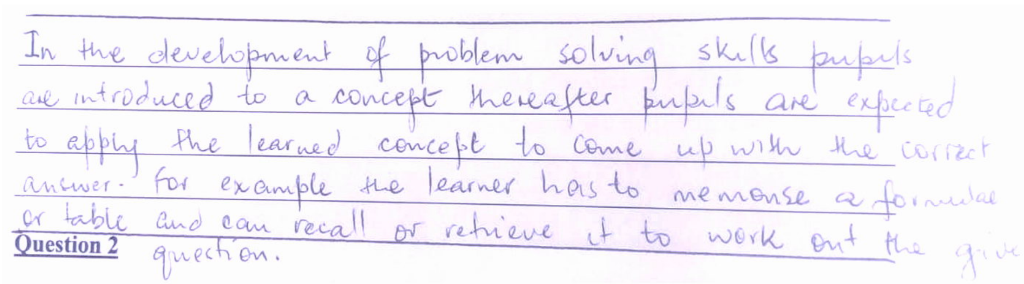
The questionnaire was administered to the participants to gain insights into their viewpoints and classroom practices and experiences on how they can help develop their learners' problem-solving skills in mathematics classrooms. The first part of the open-ended questionnaire was used to record the participants' demographic details. All participants were Basotho, and therefore, Sesotho is their first language. Other demographic details are presented in Table 1.

Table 1: Participants' demographic details

	Teacher A	Teacher B	Teacher C	Teacher D	Teacher E
Pseudonym	TA	TB	TC	TD	TE
School pseudo.	A	B	C	C	D
Gender	F	F	F	M	M
Age range	Above 40 yrs	31-35 yrs	Above 40 yrs	Above 40 yrs	Above 40 yrs
Qualifications	BSc. Ed	BSc. Ed	BSc. Ed	PGDE	BSc. Ed
Subject majors	Mathematics & Geography	Mathematics & Physics	Mathematics & Chemistry	Mathematics & Chemistry	Mathematics & Chemistry
Maths teaching experience	21 yrs and above	11-15 yrs	16-20 yrs	16-20 yrs	21 yrs and above

The participants were asked to respond to the following questions: (1) in what ways does a mathematics lesson that fosters the learners' development of problem-solving skills differ from one which does not? (2) Which methods and instructional practices do you employ, and/or classroom activities do you engage your learners to help them to develop problem-solving skills? (3) What role do you play as a teacher in the classroom to help your learners to develop problem-solving skills?

Some teachers (Teachers A and B) expressed that in a mathematical problem-solving lesson, learners are introduced to a concept after which they are given problems that allow them to use and apply the learned concept to solve similar and related problems. For example, teacher A expressed that:


In the development of problem solving skills pupils are introduced to a concept thereafter pupils are expected to apply the learned concept to come up with the correct answer. For example the learner has to memorise a formulae or table and can recall or retrieve it to work out the give question.

Question 2

Figure 2: Teacher A quotation

On the other hand, the other teachers (Teachers C, D, and E) stated that a lesson that fosters the development of the learners' problem-solving skills follows a learner-centred approach, and learners are allowed to actively engage in searching for the problem solutions on their own. For example, on the questionnaire Teacher D highlighted that:

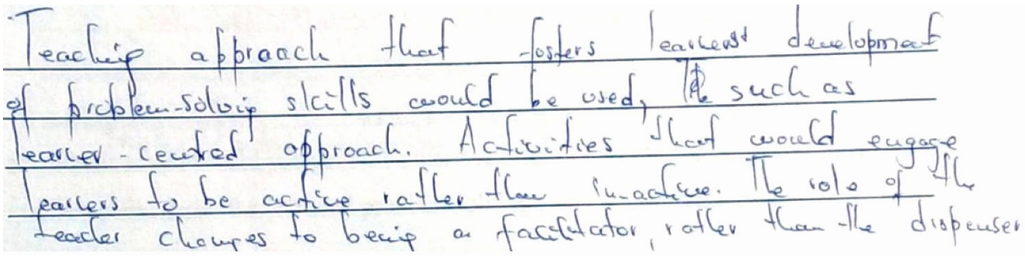

Teaching approach that fosters learners' development of problem-solving skills would be used, such as learner-centred approach. Activities that would engage learners to be active rather than inactive. The role of the teacher changes to being a facilitator, rather than the dispenser

Figure 3: Teacher D quotation

Teacher E expressed that:

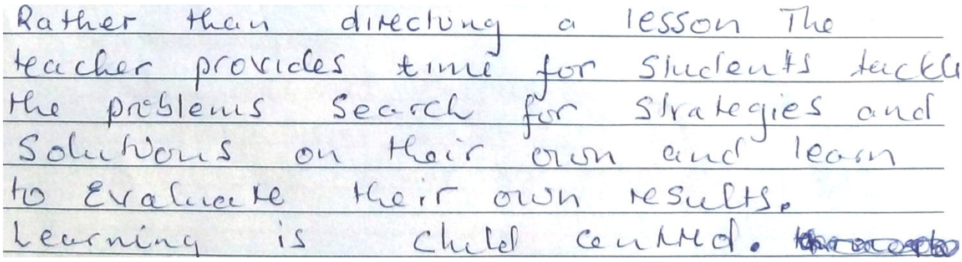

Rather than directing a lesson The teacher provides time for students tackle the problems search for strategies and solutions on their own and learn to Evaluate their own results.
Learning is child centred.

Figure 4: Teacher E quotation

The teachers also shared similar classroom practices and/or activities that engage their learners in problem-solving. The most similar ones are: giving problems to learners to solve in groups, and use of whole-class discussions where learners are encouraged to explain their solution procedures to their peers, For example, Teacher A indicated that:

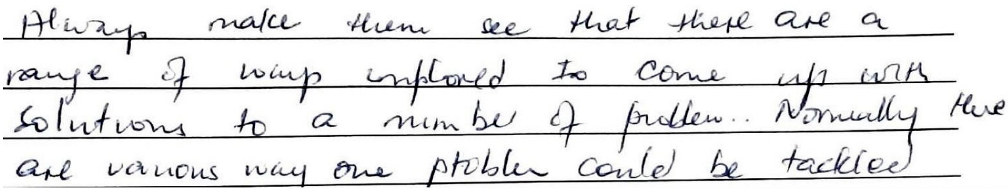

Always make them see that there are a range of ways employed to come up with solutions to a number of problem.. Normally there are various way one problem could be tackled

Figure 5: Teacher A quotation

while Teacher C stated:

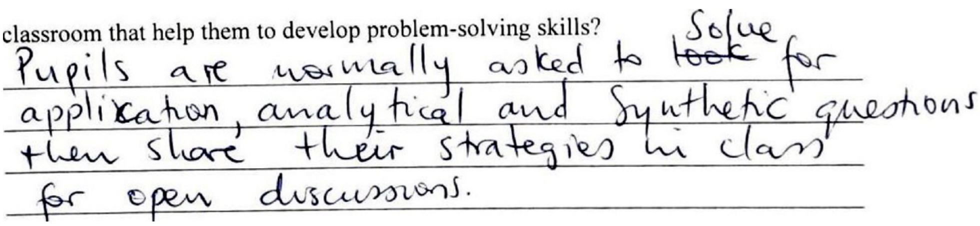

classroom that help them to develop problem-solving skills?
Pupils are normally asked to ~~look~~ solve for application, analytical and synthetic questions then share their strategies in class for open discussions.

Figure 6: Teacher C quotation

Teachers also mentioned their roles as attending to the learners' groups to guide, assist and facilitate learning through probing questions, and that they employed a wide variety of teaching methods to help learners develop problem-solving skills. For example, Teachers B and C respectively gave their teaching methods as follows.

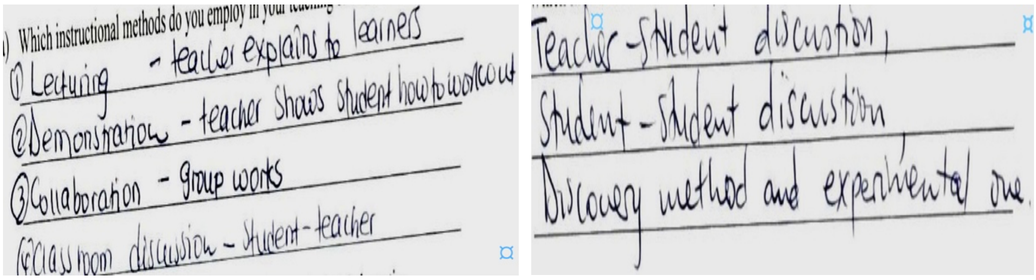


Figure 7: Teacher B quotation (left) and Teacher C quotation (right)

Findings from the observation of classroom teaching practices

The teachers were observed while teaching, to explore the teaching practices they employed in helping their learners develop problem-solving skills. The observation allowed teachers to teach topics as they had been schemed (according to the teacher's planning), and therefore, the topics taught were different and even those that were the same were taught at different times by the teachers. Each teacher was observed for three consecutive weeks and each was observed in at least 5 mathematics lessons every week. The observations focused mainly on the following classroom aspects: teaching methods, nature of the learning task/problems, classroom activities, and interactions (teacher-learners and learner-learner). Since these classroom aspects are very interrelated, the findings are grouped and presented as teaching methods and nature of the learning task/problems, and the nature of classroom activities and interactions.

1. Teaching methods and nature of the learning task/problems

The findings revealed that the teachers employed teacher-centred problem-solving teaching methods more than learner-centred, whereby lecturing, demonstration, and question-and-answer seemed to be dominating. The extract from Teacher B's lesson below illustrates the observations made.

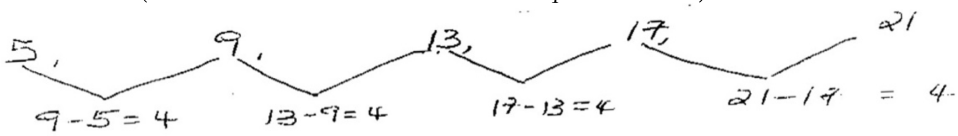
TB: Let us see (writing on the board). $^{-}5^{+4}$, $^{-}1^{+4}$, 3^{+4}, 7^{+4}, $\underline{11}^{+4,}$ $\underline{15}$. This means that the missing terms are 11 and 15. Let's now learn more about the types of sequences. Meaning if you see a sequence and analyse it, you should be able to tell that, this is this type of sequence (She wrote on the board: 5, 9, 13, 17, 21) and now let's analyse this sequence, what is its theoretical rule?

S_4 4.

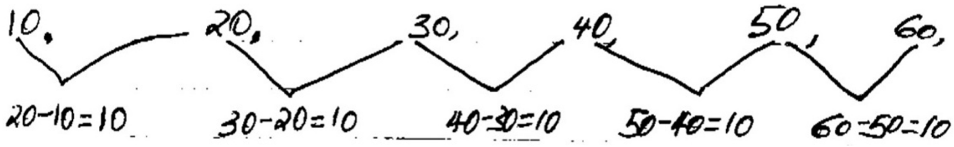
TB No! We cannot say the rule is 4.

S_5 We add 4 to the previous term to get the next term.

TB Ok let's see (she wrote on the chalkboard the sequence below)



TB Now we can see that we get the same number when we subtract the two consecutive numbers. The number we get is a common difference, so our common difference, d is equal to 4. Let us have another sequence (she wrote on the chalkboard, 10, 20, 30, 40, 50, 60). Do we have a common difference here? (she wrote on the chalkboard as follows).



TB Our d is equal to 10. So is this an arithmetic sequence?

S_{all} Yes madam.

TB Yes it is an arithmetic sequence because there is a common difference between the consecutive terms. So for arithmetic sequences, we have a specific general formula that says (she wrote on the chalkboard board) $t_n = t_1 + d(n-1)$. t_1 is…?

S_{all} Term 1.

TB How then do we use this general formula to find the n^{th} term? Let's write the same sequence.

Figures 8 and 9: Teacher B writing on chalkboard

The observational data further revealed that the teachers gave learners similar problems that were taken from the textbooks. For example, Figures 10 and 11 show the work that Teachers A and B gave to their learners.

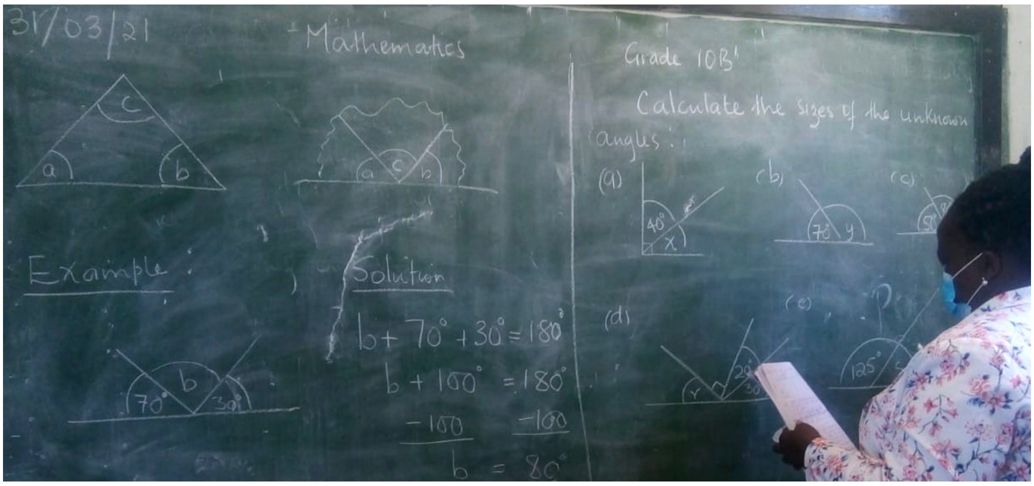


Figure 10: The work Teacher A gave to her learners

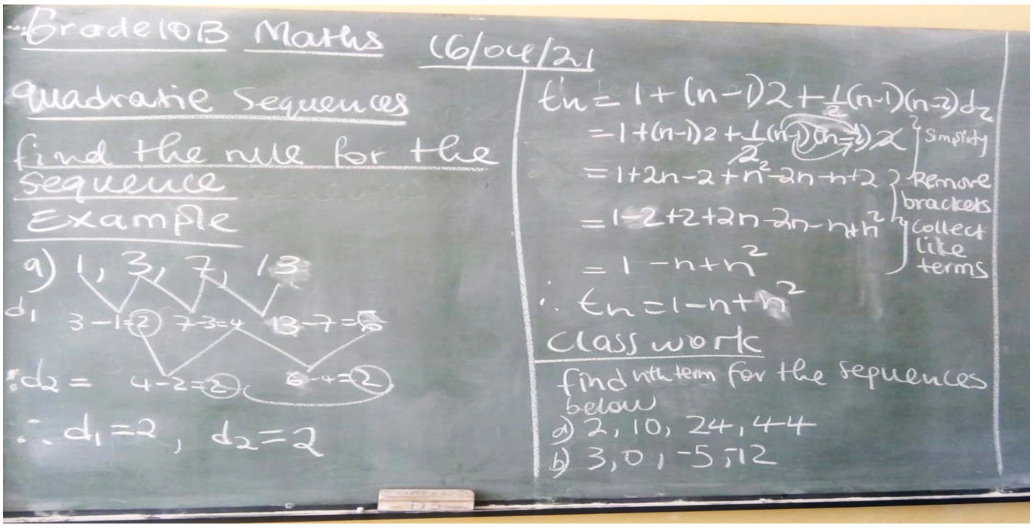


Figure 11: The work T*eacher* B gave to her learners

2. Nature of classroom interactions and activities

The findings revealed that the classroom interactions between the teacher and learners were very minimal and none among and between the learners. For example, in his lesson on Friday, 30/04/21 (at 1000-1120 hrs) Teacher D started the lesson by giving his learners a handout and the class discussed the questions from the handout.

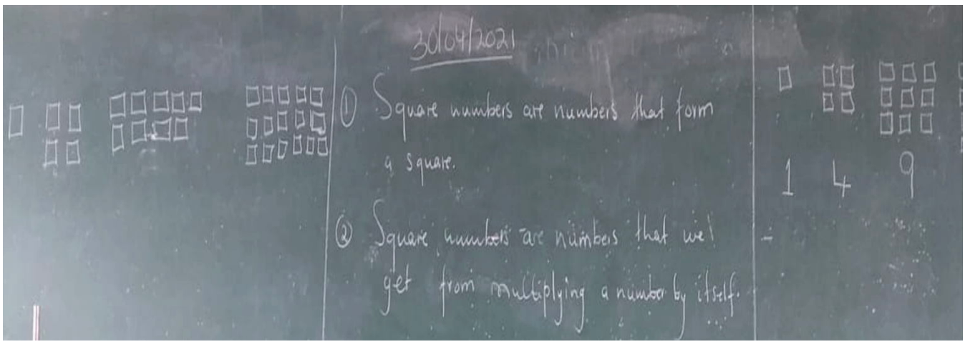
TD Question two on our handout says what are square numbers? Who wants to respond to this question?

S_1 In words Sir?

TD Yes, please!

S_2 Square numbers are numbers that form a square.

TD (Wrote this learner's response on the chalkboard as shown in picture below). *Ke mang ea nang le karabo e fapaneng?* [Anybody else with a different answer?]



S_3 Square numbers are numbers we get when multiplying numbers by themselves.

TD (Wrote the response on the chalkboard, picture above). We now have two answers from different learners. Do we agree with both answers?

S_{all} Learners shouted, others saying "the first one" whilst others said "the second one".

TD Okay, we have to respond to question number 2 by drawing it, so let us have someone draw it.

S_4 Went to the board and drew squares as shown on the left-hand side of the chalkboard, picture above.

Figure 12: Teacher D learners' responses

The findings revealed that learners' application of algorithms and formulae dominated the classroom activities. That is, before the teachers gave the learners a problem to solve, they demonstrated solution procedures and gave learners problems that require them to use similar procedures. Teachers explicitly encouraged learners to use the solution procedure they showed them. For example, the lesson extract from Teacher B's lesson (Tuesday, 11/05/21:1000-1100 hrs) went as follows.

TB Do you see how I presented the answer?

S_{all} Yes Madam (in chorus).

TB *Ke hloka hore mosebetsi oa hau u utloahale* [when you write your work you must make sure that it must have logic and make sense]. You should write the way I do so that your work is readable and has logic. I first write the actual bound (she wrote on the board 130 kg), the accuracy (she wrote 10 kg), and then the limits (she wrote 130 $+/- \frac{1}{2}$) the lower bound (she wrote LB = 130 – 5 = 125) and upper bound (she wrote UB = 130 + 5 = 135). Now I present the answer as an inequality. So our inequality is (she wrote $125 \leq 130 < 135$).

Similarly, the conversation below is from one of Teacher C's lessons (Wednesday 24/03/21: 0920-1040 hrs).

TC Since the question was not specific about whether we have to leave our answer in index form, she has to evaluate it, and this is correct. *Haeba re ne re lokela ho sia karaba e le index form?* [What if we had to leave our answer in index form?]

S_5 $(2^3)(2^2) = 2^{3+2} = 2^5$

TC Remember we add the power if the bases are the same and in most cases, we evaluate if the bases are not the same. I hope people are writing so that we have this information in our books. What if I have a d.d.d, what does it mean? Who wants to try?

The learners did not respond to her question and Teacher C shouted at them and said:
People you always have my examples in your notebooks to remind you of the work we did, why are you not referring to them instead of quietly looking at me?

Findings from the semi-structured interviews

Interviews were used to probe further into the teachers' viewpoints and classroom practices and served as a follow-up on the issues that emerged from the teachers' questionnaire responses and classroom observations. All teacher interviews were audio-recorded. The findings revealed that teachers provide their learners with problem-solving learning experiences and opportunities and those are herein presented under the two themes that emerged during data analysis: use various problem-solving teaching methods; and provide time for learners to solve problems on their own.

1. Use various problem-solving teaching methods

Probing further into the teaching methods that teachers use to help their learners develop their problem-solving skills, the teachers provided several reasons for their choices of such methods. Teacher B indicated that discussion allows learners an opportunity to share their ideas, and Teacher E added that:

> Discussion creates an atmosphere where learners interact during the lesson and get to build knowledge on their own. They share knowledge, make mistakes and correct them in trying to convince one another and this boosts their confidence and allows them to go further in learning. The discussion also helps me to monitor their progress. Ahh, we need to be able to detect misconceptions and deal with them right away in guiding them to develop the desired skills.

Teacher D also supported that

> During class discussions, I ask my learners to share their solutions on the chalkboard so that we see how they got them. The solutions presented on the chalkboard are discussed, peers share ideas, check whether the answer and procedure used are correct if not, identify errors/mistakes, and show the correct way of solving the problem.

However, teachers warned about a challenge with the use of large groups that some students don't participate in and hide behind others, and Teachers B and C expressed they prefer small groups.

> I prefer groups of two or three pairs, rather than in large groups. This easily allows me to identify those who are not participating, but if the group is large then it's not possible (Teacher B).

Some teachers expressed that they sometimes lecture learners, and use question-and-answer and demonstrations in cases where they find that students do not have a good background knowledge of the lesson topic making it difficult for learners to discuss (Teachers B and C). Teacher C further stated that question-and-answer and demonstration methods help develop learners' ability to ask questions and explain whenever asked to do so. On the other hand, Teacher E stated that he prefers the investigation method because it allows learners to explore and find knowledge of the concepts and even beyond. He concluded that "Investigation allows learners a chance to present information in a way that they understand it".

2. Provide time for learners to solve problems on their own

Participants highlighted that developing learners' mathematical problem-solving skills requires allowing learners an opportunity to explore and solve problems on their own. For example, Teacher D expressed that

> We should give learners time whenever we give them a problem to solve. They should be the ones coming up with different ways of solving the problems, rather than getting or listing to the teacher all the time telling them the steps to follow. That is learners should somehow interact because they turn to do better when they are with others. I also think that we should have a classroom situation that makes learners hands-on, and interact with one another. I don't know, but our classroom situations should give such ahh ... time or whatever.

However, teachers seemed to be challenged to implement problem-solving in their teaching of mathematics. They highlighted that problem-solving requires a lot of time while they need to finish teaching syllabus content and they are challenged in designing problem-solving questions. For example:

TB ... you will find that I am pressed by time, as there is a lot for these learners to learn, there is a lot to teach on the plate.

TD I think this is not possible to cover the syllabus content and develop learners' problem-solving skills because that itself requires a lot of time.

TC *[On the other hand]*
... my main challenge is that the learners do not have textbooks and this makes it difficult for me because it is not easy for us as teachers to formulate our problem-solving questions.

In responding to the follow-up question on the role and assistance they give to learners during problem-solving the participants stressed the importance of the teacher's presence to facilitate their development of problem-solving skills. Teachers C and D stated their views as follows.

TC I move around and present myself to the groups to allow them to ask questions and to probe and guide them. The teacher's availability for learners during problem-solving is an important aspect as far as I am concerned.

TD We should be facilitators, but we normally turn to make it all about us telling them what to do. We should do away with traditional teaching whereby the teacher is the one who dispenses knowledge, and learners just are passive.

Discussion

All the participants seemed to possess relevant knowledge of the classroom practices and activities that foster, nurture, develop, and/or enhance learners' problem-solving in mathematics classrooms. The teachers mentioned two important points in developing problem-solving skills that are similar to those mentioned in the literature: (1) allow learners time and an opportunity to engage with mathematical problems on their own whereby the teacher is the facilitator, probing and guiding learners, instead of telling and/or showing them how they should solve the problems (Evans, 2012; NCTM, 2014); (2) engaging learners in a peers discussion to foster their participation, to argue their mathematical ideas and thinking and share knowledge (Stein, Grover & Henningsen, 1996).

However, teachers seemed to be challenged in employing classroom practices and activities that allow learners opportunities to develop problem-solving skills. That is, their use of teacher-directed problem-solving teaching methods (like lecturing and demonstration of the solution procedures using mathematical problems taken from the textbooks), and minimum classroom interactions were the main drawbacks. This observation also came from the teachers themselves when they stated that problem-solving requires time to teach, yet there is a lot of syllabus content they have to teach before learners sit for an end-of-year or external examinations. This implies that mathematics teaching in classrooms is still content-focused and examinations-driven.

Allowing learners an opportunity and time to explore and try out problems using their solution procedures not only helps them gain a deeper mathematical understanding but also makes them recognise their ability as independent thinkers. Equally important, peers are very important in one's problem-solving development. Allowing learners to share their solution strategies on the chalkboard should also encourage them to request clarity, challenge, or counteract the presented solutions procedures. This will make learners aware that mathematics learning is not just about getting the correct answer, but rather the individual's logical thinking behind such an answer. Demonstrating solution answers may make learners believe that mathematics is best learned through a lot of practice on memorised facts, procedure, and algorithms. It denies them an opportunity to consider themselves as potential problem-solvers, fast thinkers and risk-takers.

A framework to assist the teachers to integrate problem-solving

This section presents an analysis as well as a synthesis of the data on the teachers' viewpoints and the classroom practices that help develop learners' mathematical problem-solving skills. The key ideas identified for the development of a framework are presented in Figure 13.

Constructs of the proposed framework

Based on the teachers' perspectives and classroom practices, it is proposed that the framework should be centred on two main constructs in developing learners' problem-solving skills, namely the learning task and the learning environment. That is, the teacher should develop authentic, rich, and meaningful problems for students and create a conducive and supportive learning environment for learners to develop their potential in problem-solving.

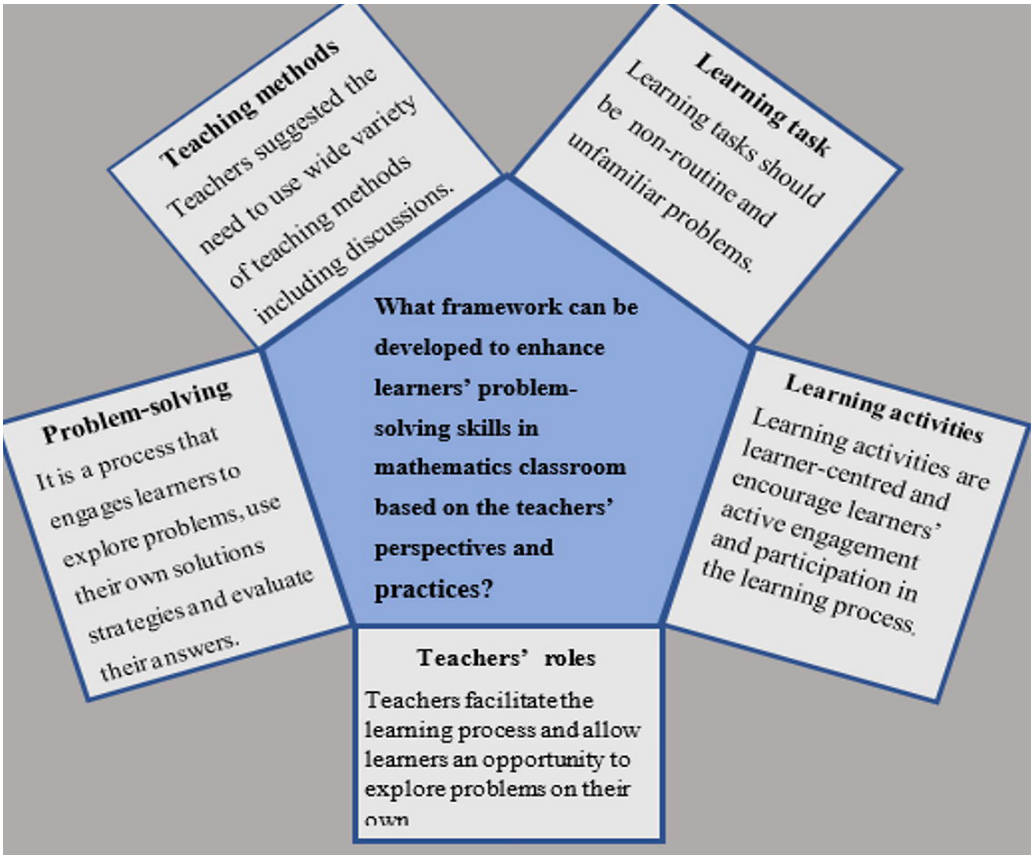


Figure 13: Key ideas identified from the teachers' perspectives and classroom practices on problem-solving to be used in developing the framework

Learning task

The teachers should provide learners with a novel, rich and meaningful task to discover the understanding of mathematics concepts and develop their problem-solving skills. That is, the teacher should give learners the main idea, and allow them time and an opportunity to explore and get the details from such a main idea (Aljohani, 2017).

Classroom learning environment

The findings suggest that individual classroom teachers should reflect critically upon their problem-solving practices and mathematics-related beliefs (its nature, teaching, and learning), and adapt their teaching to embrace a classroom learning environment that creates opportunities and offers them learning experiences that support engagement in rich mathematical interactions to develop their problem-solving skills, and hence deeper mathematical understanding. In such a learning environment, the teachers reclaim their roles in the learning process as the facilitators and hence, consider learners as capable independent, autonomous thinkers and potential problem-solvers.

Presentation of the framework

This section presents the framework developed through an analysis of the teachers' viewpoints and practices that assist them to develop learners' problem-solving skills in mathematics (Figure 14). This section responds directly to the research question: *"What framework can be developed to enhance learners' problem-solving skills in a mathematics classroom based on the teachers' perspectives and practices?"*

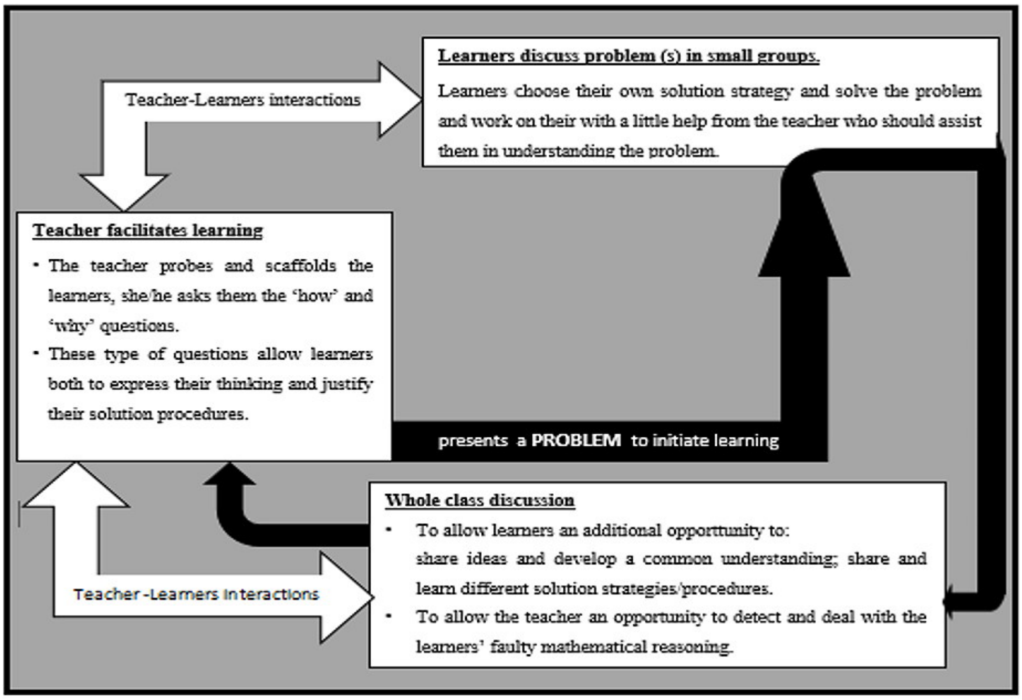


Figure 14: The developed framework

The framework works in a circular manner whereby the teacher first presents a problem to the learners and then allows time for them to discuss the problem(s) in small groups. Finally, the teacher brings learners together for whole-class discussions. Each of these key constructs of the framework is discussed below.

The teacher sets the learning stage

This model is founded on the understanding that the student's discovery and construction of knowledge are dependent on the support from teacher-learner and learner-learner interactions. Therefore, the teacher should set the stage for learning and activate learners' prior knowledge. This means that the teacher's primary role is to facilitate the students' construction of their knowledge and understanding. A teacher must present a problem, a learning task, to the learners to initiate the learning process. This framework emphasises that the learning task should be an experience through which learners explore and meaningfully learn mathematics, and therefore, must be presented to initiate students' thinking and engage them in the learning process. This contradicts the practice in a traditional classroom, whereby the teacher uses the problem to explain mathematical facts/concepts and demonstrate the solution procedure for learners to memorise and apply in solving similar problems.

Learners' discussion of problem(s) in small groups
Having presented the problem, the teacher should allow learners sufficient time to engage with and explore the problem(s) in small groups. This is meant to allow learners to share views and understandings and hence learn from one another on their own. It is in this stage where the teacher is a facilitator, guiding learners in following steps in solving unfamiliar problems as suggested by Polya (1945) and Schoenfeld (1985) and probing them through the use of higher-order questions such as 'how' and 'why', to encourage learners to communicate about and reflect on their mathematical thinking.

Whole class discussions
The teacher then brings learners together for the whole class discussion to give them yet another opportunity to learn from one another, share ideas and build a common understanding. It is at this moment that learners are allowed an opportunity to present and share their different solution strategies on the chalkboard for peers to critique, argue, improve and most importantly, learn from. This is in the stage where the teachers' best questioning and facilitation skills are critical to relating and bringing together the learners' divergent thinking, to attain a common mathematical understanding.

Conclusions

This framework is based on the teachers' viewpoints and practices that were expressed and observed in this study. It is envisaged that it can help mathematics teachers to integrate problem-solving into their classroom activities, both in Lesotho where this study was conducted and other parts of the world. Mathematics teachers can therefore adapt their classroom practices to assist learners to develop their mathematics thinking, knowledge, and application through solving problems.

Recommendations

The relevant government bodies and offices should capacitate and assist schools in organising their own in-house professional workshops, training, and/or collaborative discussions to create platforms that aim to help teachers to reflect on their beliefs and knowledge on problem-solving, and on the nature of mathematics, its teaching and learning. These activities should also help teachers to design real mathematical problems, and gain a deeper understanding of the relationships between and amongst the mathematical facts and concepts. They can then focus more on big mathematical concepts that can help them to direct their teaching more towards developing learners' mathematical conceptual understanding. This will minimise the disconnection between mathematical concepts brought by the 'teaching of the syllabus named mathematical topics', that seem to put the teachers under pressure and frustration that there is a lot of curriculum content they are expected to cover.

References

Doorman, M., Drijvers, P., Dekker, T., van den Heuvel-Panhuizen, M., de Lange, J. & Wijers, M. (2007). Problem-solving as a challenges for mathematics education in The Netherlands. *ZDM Mathematics Education,* 39(5-6), 405-418. https://doi.org/10.1007/s11858-007-0043-2

Akinmola, E. A. (2014). Developing mathematical problem-solving ability: A panacea for a sustainable development in the 21st Century. *International Journal of Education and Research,* 2(2), 1-8. https://www.ijern.com/journal/February-2014/28.pdf

Aljaberi, N. M. & Gheith, E. (2016). Pre-service class teachers' ability in solving mathematical problems and skills in solving daily problems. *Higher Education Studies,* 6(3), 32-47. https://doi.org/10.5539/hes.v6n3p32

Alhojailan, M. I. (2012). Thematic analysis: A critical review of its process and evaluation. *West East Journal of Social Sciences,* 1(1), 39-47. https://faculty.ksu.edu.sa/sites/default/files/ta_thematic_analysis_dr_mohammed_alhojailan.pdf

Aljohani, M. (2017). Principles of "constructivism" in foreign language teaching. *Journal of Literature and Arts studies,* 7(1), 97-107. https://doi.org/10.17265/2159-5836/2017.01.013

Anderson, J. & White, P. (2004). Problem solving in learning and teaching mathematics. In B. Perry, G. Anthony & C. Diezmann (Eds.), *Research in mathematics education in Australasia: 2000-2003* (pp. 127-150). Flaxton: QLD: PostPressed.

Artigue, A., Haspekian, M. V. & Corblin-Lenfant, A. (2014). Introduction to the theory of didactical situation (TDS). In A. Birker-Ahsbahs & S. Prediger (Eds.), *Networking of theories as a research practice in mathematics education* (pp. 47-65). Springer. https://doi.org/10.1007/978-3-319-05389-9_4

Atteh, E., Andam, E. A. & Obeng-Denteh. (2017). Problem solving framework for mathematics discipline. *Asian Research Journal of Mathematics,* 4(4), 1-11. https://doi.org/10.9734/ARJOM/2017/32586

Bishaw, A. (2010). Teachers' beliefs and actual practices of problem solving approach in teaching mathematics (with particular reference to Grades 9 and 10 in West Gojjam). *Ethopian Journal of Education and Sciences,* 6(1), 73-87. https://doi.org/10.4314/ejesc.v6i1.65382

Bostic, J. D., Pape, S. J. & Jacobbe, T. (2016). Encouraging sixth-grade students' problem-solving performance by teaching through problem solving. *Investigations in Mathematics Learning,* 8(3), 30-58. https://doi.org/10.1080/24727466.2016.11790353

Bradshaw, Z. & Hazell, A. (2017). Developing problem-solving skills in mathematics: A lesson study. *International Journal for Lesson and Learning Studies,* 6(1), 32-44. https://doi.org/10.1108/IJLLS-09-2016-0032

Braun, V. & Clarke, V. (2006). Using thermatic analysis in psychology. *Qualitative Research in Psychology,* 3(2), 77-101. https://doi.org/10.1191/1478088706qp063oa

Braun, V., Clarke, V. & Weather, P. (2016). Using thermatic analysis in sport and exercise research. In B. Smith & A. C. Sparkes (Eds.), *Handbook of qualitative research in sports and exercise* (pp. 191-205). London: Routledge. https://www.taylorfrancis.com/chapters/edit/10.4324/9781315762012-26/using-thematic-analysis-sport-exercise-research-virginia-braun-victoria-clarke-paul-weate

Brousseau, G. (1977). *Theory of didactical situations in mathematics: Didactique des Mathématiques, 1970-1990*. Boston: Kluwer Academic Publishers. https://doi.org/10.1007/0-306-47211-2

Căprioară, D. (2015). Problem solving- purpose and means of learning mathematics in school. *Procedia - Social and Behavioral Sciences,* 191(2015), 1859-1864. https://doi.org/10.1016/j.sbspro.2015.04.332

Chapman, O. (2015). Mathematics teachers' knowledge for teaching problem-solving. *LUMAT: International Journal on Math, Science and Technology Education,* 3(1), 19-36. https://doi.org/10.31129/lumat.v3i1.1049

Cope, D. G. (2014). Methods and meanings: Credibility and trustworthiness of qualitative research. *Oncology Nursing Forum,* 41(1), 89-91. https://doi.org/10.1188/14.ONF.89-91

Creswell, J. W. (2013). *Qualitative inquiry and research design: Choosing among five approaches* (3rd ed.). USA: SAGE Publishing. [4th ed.] https://us.sagepub.com/en-us/nam/qualitative-inquiry-and-research-design/book246896

Cypress, B. S. (2017). Rigor or reliability and validity in qualitative research: Perspectives, strategies, reconceptualization and recommendations. *Dimensions of Critical Care Nursing,* 36(4), 253-263. https://doi.org/10.1097/DCC.0000000000000253

Davis, R., Maher, C. A. & Noddings, N. (1990). Introduction: Constructivist views on the teaching and learning of mathematics. In R. Davis, C. Maher & N. Noddings (Eds.), *Constructivist views on the teaching and learning of mathematics* (pp. 7-18). Reston, VA: National Council of Teachers of Mathematics. https://doi.org/10.2307/749908

Delacour, L. (2012). Interpreting the curriculum - mathematics and didactic contracts in Swedish preschools. *Utbildning & Lärande,* 6(2), 64-79. http://www.diva-portal.org/smash/get/diva2:1256597/FULLTEXT01.pdf

DiMatteo, R. W. & Lester, Jr. F. K. (2010). The role of problem solving in secondary school mathematics classroom. In J. Lobato & F. Lester (Ed.), *Teaching and learning mathematics: Translating research for secondary school teachers* (pp. 7-12). Reston: VA: NCTM. https://www.nctm.org/Store/Products/Teaching-and-Learning-Mathematics--Translating-Research-for-Secondary-School-Teachers-(Download)/

Ernest, P. (1989). Philosophy, mathematics and education. *International Journal of Mathematical Education in Science and Technology,* 20(4), 555-559. https://doi.org/10.1080/0020739890200409

Evans, B. R. (2012). Editor's perspective article: Problem-solving abilities and perceptions in alternative certification mathematics teachers. *JNAAC,* 7(2), 34-43. https://files.eric.ed.gov/fulltext/EJ1053509.pdf

Govender, V. G. (2015). What are the views of mathematics teachers regarding implementation of problem solving in their classrooms? A study involving both primary and secondary teachers. In *Deepening the quality of mathematics teaching and learning. Proceedings of the 21st Annual national congress of the Association for Mathematics Education of South Africa* (pp. 184-204). Polokoane-Limpopo Province: AMESA.

Guzman Gurat, M. (2018). Mathematical problem-solving heuristics among student teachers. *Journal of Efficiency and Responsibility in Education and Science,* 11(3), 53-64. https://files.eric.ed.gov/fulltext/EJ1208772.pdf [also https://doi.org/10.7160/eriesj.2018.110302]

Harisman, Y., Kusumah, Y. S. & Kusnandi, K. (2019). Beliefs of junior high school teachers on learning process on mathematical problem solving. *Journal of Physics: Conference Series,* 1157(3) article 032112. https://doi.org//10.1088/1742-6596/1157/3/032112

Hwang, J. & Riccomini, P. J. (2016). Enhancing mathematical problem solving for secondary students with or at risk of learning disabilities: A literature review. *Learning Disabilities Research & Practice,* 31(3), 169-181. https://doi.org/10.1111/ldrp.12105

Mohammed, S. H. & Kinyo, L. (2020). The role of constructivism in the enhancement of social studies education. *Journal of Critical Reviews,* 7(7), 249-256. https://publicatio.bibl.u-szeged.hu/25153/1/JCR_Saif_Mohammed_Laszlo_Kinyo.pdf

Matić, L. J. (2017). The effects of problem solving course on pre-service teachers' beliefs about problem solving in school mathematics and themselves as problem solvers. *Magistra Iadertina,* 12(2), 134-152. https://doi.org/10.15291/magistra.1494

McIntosh, R. & Jarret, J. (2000). *Teaching mathematical problem solving: Implementing the vision.* Oregon: Mathematics and Science Education Center. http://academic.sun.ac.za/mathed/174WG/Teaching%20Problemsolving.pdf

Minarni, B. M., Retnawati, H. & Nugraheni, T. V. T. (2018). Mathematics teacher's belief and its contribution towards teaching practice and student achievement. *Journal of Physics: Conference series,* volume 1097, article 012143. https://doi.org/10.1088/1742-6596/1097/1/012143

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics.* Reston, VA: National Council of Teachers of Mathematics (NCTM). https://www.nctm.org/standards/

National Council of Teachers of Mathematics (2014). *Principles to actions: Ensuring mathematical success for all.* Reston: VA: National Council of Teachers of Mathematics (NCTM). https://www.nctm.org/Store/Products/Principles-to-Actions--Ensuring-Mathematical-Success-for-All/

Norton, S., McRobbie, C. J. & Cooper, T. J. (2002). Teachers' responses to an investigative mathematics syllabus: Their goals and practices. *Mathematics Education Research Journal,* 14(1), 37-59. https://doi.org/10.1007/BF03217115

Osman, S., Che Yang, C. N. A., Abu, M. S., Ismail, N., Jambari, H. & Kumar, J. A. (2018). Enhancing students' mathematical problem-solving skills through bar model visualization technique. *International Electronic Journal of Mathematics Education,* 13(3), 273-279. https://doi.org/10.12973/iejme/3919

Polya, G. (1945). *How to solve it?* Princeton, NJ: Princeton University Press.

Radford, L. (2018). On theories in mathematics education and their conceptual differences. In B. Sirakov, P. de Souza & M. Viana (Eds.), *Proceedings of the International Congress of Mathematics* (pp. 4037-4055). Singapore: World Scientific Publishing Co. Vol 4. https://doi.org/10.1142/9789813272880_0211

Rott, B. (2012). Models of the problem solving process - a discussion referring to the processes of fifth graders. In T. Bergqvist (Ed.), *Learning problem solving and learning through problem solving, Proceedings from the 13th ProMath Conference, September 2-4, 2011* (pp. 95-105). Umea, Sweden: Umea University Faculty of Sciences and Technology, Umea Mathematics Education Research Centre. http://docplayer.net/24733110-Problem-solving-in-mathematics-education-proceedings-from-the-13-th-promath-conference-september-2-4-2011-in-umea-sweden.html

Schoenfeld, A. H. (1985). *Mathematical problem solving.* New York: Academic Press.[see https://www.routledge.com/Mathematical-Thinking-and-Problem-Solving/Schoenfeld-Sloane/p/book/9780805809909]

Shenton, A. K. (2004). Strategies for ensuring trustworthiness in qualitative research project. *Education for Information,* 22(2), 63-75. https://content.iospress.com/articles/education-for-information/efi00778

Stein, M. K., Grover, B. W. & Henningsen, M. (1996). Building student capacity for mathematical thinking and reasoning: An analysis of mathematical tasks used in reform classrooms. *American Educational Research Journal*, 33(2), 455-488. https://doi.org/10.3102/00028312033002455

Stenfors, T., Kajamaa, A. & Bennett, D. (2020). How to ... assess the quality of qualitative research. *The Clinical Teacher,* 17(6), 596-599. https://doi.org/10.1111/tct.13242

Tracy, S. J. & Hinrichs, M. M. (2017). Big tent criteria for qualitative quality. In J. Matthes (Ed.), *The international encyclopedia of communication research methods*, 1-10. Wiley. https://doi.org/10.1002/9781118901731.iecrm0016

Uys, D. (2010). *The function of teachers' code switching in multilingual and multicultural high school classrooms in the Siyanda district of the Northern Cape province*. Masters thesis, Stellenbosch University, South Africa. http://hdl.handle.net/10019.1/4361

Zakaria, E. & Maat, S. M. (2012). Mathematics teachers' beliefs and teaching practices. *Journal of Mathematics and Statistics,* 8(2), 191-194. https://thescipub.com/pdf/jmssp.2012.191.194.pdf

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Please cite as: Malepa-Qhobela, M. & Mosimege, M. (2022). A framework to assist mathematics teachers in integrating problem solving in secondary school classrooms. *Issues in Educational Research*, 32(4), 1486-1508. http://www.iier.org.au/iier32/malepa-qhobela.pdf