Using universal design for instruction principles to guide flexible, inclusive and accessible teaching of geometry

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When geometry teaching lacks elements of flexibility, inclusivity and accessibility, it is often difficult for learners to follow and understand what they are being taught. In line with this, this paper reports on the application of universal design for instruction (UDI) to guide flexible and accessible teaching of geometry to maximise learning. UDI is an approach to teaching that uses a variety of flexible, inclusive and accessible teaching methods to remove barriers to learning, in order to give all learners equal opportunity to succeed. Eight grade 9 mathematics teachers in one of the largest secondary schools in the Motheo district in South Africa were purposively selected to participate in this qualitative case study that was underpinned by sensemaking theory as a lens. These teachers have previously undergone training on the application of UDI in teaching mathematics. Focus group discussions and lesson observations were used to generate data, whilst content analysis was used to analyse the generated data. The findings of the study revealed the positive effects of UDI in terms of guiding flexible, inclusive and accessible teaching of geometry to maximise learning. Thus, this study suggests implications for teaching geometry using UDI to maximise learning.

Introduction

Geometry is one of the most crucial topics in mathematics (Dimakos & Nikoloudakis, 2009). It is one of the topics that is outlined in the South African mathematics curriculum namely, curriculum assessment policy statement (CAPS). It is a branch of mathematics which deals with properties such as shapes of figures, lines, angles, circles, etc. (Brannan, Esplen & Gray, 2002). According to Mamali (2015: p.6), "geometry also involves the study of position of the objects, their movements and the space around them". Geometry is regarded by Güven and Kosa (2008) as the study of space and shape which requires logical reasoning skills in order for the related problems to be solved. In the context of this study, geometry is defined as a branch of mathematics which deals with properties, such as shapes of figures, lines, angles, circles, etc. (Brannen et al., 2002).

Clements and Battista (1992) maintained that knowledge and understanding of geometry is significant because the world in which we live is "inherently geometric", meaning that the world consists of objects which are constructed in the form of shapes. Knowledge of geometry further assists in developing learners' logical reasoning skills that is needed not only for solving geometry riders but also required in other fields of study wherein logical reasoning skills are used, such as architecture, civil engineering, and building and construction (Alex & Mammen, 2018; Ubah & Bansilal, 2019; Sergeeva, 2020). From these, it can be deduced that geometry serves as a prerequisite in other fields of study and thus necessitates that it should be taught well so that learners may acquire a strong foundation to be able to apply it in future (i.e. in their chosen fields of studies where it serves as a prerequisite).

Several authors (Adolphus, 2011; Chiwiye; 2013; Mashingaidze, 2012) have attributed the reasons for poor performance in geometry to unproductive traditional teaching methods that are predominantly used in mathematics classrooms. These unproductive teaching methods promote rote learning (Tsimane, 2020) and limit the learners' conceptual understanding (Viennot, 2008). Furthermore, these unproductive traditional teaching methods do not promote deep learning of mathematical concepts and often result in the poor development of learner reasoning skills (Umugiraneza, Bansilal & North, 2017). Unproductive traditional methods are often characterised by the transmission of knowledge from the teacher to the learners (Dejene, Bishaw & Dagnew, 2018). Unproductive traditional methods disregard the fact that learners have diverse needs, the very core emphasis about learner diversity being an issue that is worth addressing in education practices (Possi & Milinga, 2017). They also deprive learners of opportunities to discover "concepts" by themselves. In line with this, Freudenthal (1973) noted that discoveries that are made by one's eyes and by being engaged in hands-on activities are vital for meaningful learning and for this reason, the use of flexible, inclusive and accessible approaches in the teaching and learning of geometry should be regarded as important and worthy of exploration.

Adolphus (2011) noted that learners find geometry concepts difficult to understand and to solve. Mamiala, Mji and Simelane-Mnisi (2021, p.487) postulated that "the major contributors of challenges pertaining to the teaching and learning of geometry relate to teaching pedagogy, teaching methodologies, students' lack of interest in geometry, lack of understanding of many geometric concepts and lack of use of technology". To attempt to address the challenges pertaining to the teaching and learning of geometry, previous research has highlighted several strategies that have been implemented. For example, teaching geometry using the lens of the theory of geometric working spaces (Kuzniak, 2018), using *Minecraft* to teach spatial geometry (Foerster, 2017), using augmented reality tools and materials for teaching geometry (İbili, Çat, Resnyansky, Şahin & Billinghurst, 2020), using *Geogebra* software and automated reasoning tools (Onaifoh & Ekwueme, 2017), the application of ethno-mathematical approaches (Verner, Massarwe & Bshouty, 2019), and using manipulatives (Hidayah, 2018), to highlight but a few.

Although the above strategies have attempted to improve the teaching and learning of geometry, teachers still find it difficult to teach the topic, due to a lack of content knowledge as well as an understanding of strategies to teach geometry as a practical topic (Mashingaidze, 2012). Teachers therefore seem to be unable to teach geometry in a flexible, inclusive and accessible manner, which makes it difficult for learners to follow and understand what they are being taught, and for their needs to be catered for in terms of the strategies mentioned above. In line with this, Jones (2002) emphasised the need for 'appropriate' efforts to be made in terms of teaching geometry since this can maximise learning and result in improved performance in geometry and mathematics in general. On the basis of this therefore, the current study reports on the application of *universal design for instruction* (UDI) to guide flexible, inclusive and accessible teaching of geometry to maximise learning. The study thus responds to the following question:

> How can the universal design for instruction (UDI) be used to guide flexible, inclusive and accessible teaching of geometry to maximise learning?

Universal design for instruction

Universal design for learning (UDL) is an approach to teaching that comprises the proactive design and use of inclusive instructional strategies that benefit a broad range of learners including students with disabilities (Scott, McGuire & Embry, 2002). It is a teaching framework that guides flexible, accessible and inclusive teaching (Moleko & Mosimege, 2020) and it is aimed at eradicating barriers from the learning environment (Moleko, 2018). Many people often carry the misconception that UDL is exclusively meant for teaching learners with disabilities. However, according to Boothe (2018) and Dalton (2017), it is a framework that is meant for teaching all learners regardless of whether they have disabilities or not. A broader framework of teaching which encompasses the best practices and principles is a *universal design for instruction* (UDI) framework which is incorporated within the broader framework of UDL (Zaloudek, 2014).

The UDI framework was coined by the DO-IT's Center for Universal Design in Education (CUDE) at the University of Washington. The definition that is used for its application is adapted from the basic definition of universal design, a concept used in the field of architecture. UDI refers to 'the design of teaching and learning products and environments that are usable by all learners to the greatest extent possible, without the need for adaptation or specialised design' (Shaw, 2011). The UDI framework is intended to ensure that all aspects of teaching and learning (e.g. physical spaces, curriculum, pedagogy, and technology) are flexible, accessible and inclusive (Burgstahler, 2009).

UDI entails the awareness of learner diversity, the expectation of a variety of learning needs, and an intentional approach to designing inclusive, flexible and accessible learning spaces (Adaptive Environments Center Inc., 2000; Center for Universal Design, 1997; Covington & Hannah, 1997). UDI simply encourages teachers to anticipate the diversity of learners in their classrooms and necessitates that they proactively build in approaches to learning and assessment that are more flexible, inclusive and accessible to a broader range of learning needs (van Jaarsveldt & Ndeya-Ndereya, 2015). The notion of proactively designing teaching that is responsive to diversity in learning is worthy of further exploration (Scott et al., 2003). The UDI framework comprises nine principles which encompass guidelines for devising teaching that is flexible, inclusive and accessible as shown in Appendix 1.

Applicability of UDI in the teaching of geometry

The fact that learners learn and assimilate information differently requires teachers to vary their teaching strategies to accommodate a diverse learner population. Geometry is regarded as a practical topic, which many learners find difficult to learn. This, therefore, means that teachers need to make efforts in terms of devising practical strategies that are flexible, accessible and inclusive in order to ensure that they meet the needs of the learners. The UDI framework therefore encompasses guidelines that will help inform flexible, accessible and inclusive teaching strategies to maximise learning of geometry concepts.

Significance of the study

This study has at least two theoretical inferences. First, the findings contribute to the advancement of the theories and principles of teaching geometry by using UDI as an approach for guiding flexible, accessible and inclusive teaching of geometry. Second, this study highlights implications for teaching geometry using UDI to maximise learning, thus guiding the design of flexible, accessible and inclusive geometry lessons.

The implications of the present study are therefore positioned within the debates on promoting inclusive education to enhance the teaching and learning of mathematics, as underscored in the results of some international benchmark assessments such as *Trends in International Mathematics and Science Study* (TIMSS, undated).

Sensemaking theory as the lens underpinning the study

Karl Weick's sensemaking theory was adopted as a lens couching this study in order to afford the participants opportunities to engage in the process of meaning making (McNamara, 2015). Sensemaking is an interpretative, explanatory and knowledge-creating process which is based on interpretations, decisions, and individual perceptions (Coetzee & Wilkinson, 2020). The sensemaking theory encompasses seven phases namely, identity; social activities; enactment; cues; plausibility; retrospection, and ongoing processes (Weick, 1995). In this study, a focus was placed on the retrospective phase in which the participants reflected on their previous teaching experiences, thus scrutinising their decisions and experiences in order to make meaning of their teaching practices (Perryman, 2011; Berberich, 2016).

Sensemaking requires the schools to be viewed as organisations that exemplify flexible, accessible and inclusive teaching as one of the school's important activities. According to Langenberg and Wesseling (2016) the sensemaking lens helps in terms of bridging the gap between theory and practice. The sensemaking theory was adopted in this study to afford the study's participants an opportunity to give meaning to their geometry teaching experiences and to further afford the researcher an opportunity to explore how the participants give meaning to their practices and experiences (McNamara, 2015). The participants in this study were given a platform to share their experiences of applying the UDI framework in the teaching of geometry. This subsequently gave them an opportunity to give meaning and make sense of their practices.

Method

This was a phenomenological case study that involved one large high school in the metropolitan city in Motheo district, Free State province (the most central province in South Africa). The school comprised 1600 learners. A purposive sampling technique was used to select the school in which eight mathematics teachers (5 senior phase and 3 FET phase) were involved in the research project. Although English is the main language used in the classroom for teaching and learning and assessments, teachers sometimes "code switch" by using the students' home language (Sesotho in the context of this study) to clarify some mathematical concepts and to reinforce understanding. All mathematics teachers went through the UDI hands-on / practical workshop in 2019. The workshop was meant to equip teachers with knowledge and skills on how to apply the UDI principles in a mathematics classroom. The school caters for grade 7 to grade 12 learners.

The participants of this study were secondary school mathematics teachers (Further Education Training, FET) (grades 10 to 12) who also taught in the senior phase (grades 7 to 9). In South Africa, the grades (previously known as standards) are categorised according to phases. For example, Grades R to 3 (pre-school, Sub A, Sub B and standard 1) are called foundation phase; grades 4 to 6 (standards 2 to 4) are called intermediate phase; grades 7 to 9 (standards 5 to 7) are called senior phase and grades 10 to 12 (standards 8 to 10) FET phase. Data were generated through three focus group discussions (FGDs) (each session lasting for 1 hour), in which all the participants (8 teachers) had attended. There were also five class observation sessions, each session lasting two hours. The FGDs and observations took place from 1 to 10 April 2019 during school-working hours (Monday to Friday). During the first five days (1 to 5 April 2019), observations were conducted followed by the focus group discussions. All observed lessons and FDGs were video recorded. The free attitude interview (FAI) technique (Meulenberg-Buskens, 2011) was used to facilitate the FDGs. This technique requires one comprehensive research question to be asked to initiate the discussion. In line with this, the following question was asked to initiate the discussion:

> How can the UDI principles be implemented to guide flexible, inclusive and accessible teaching of geometry?

During the discussions the researcher asked for clarity and redirected the question(s) (emanating from the discussion) where necessary to ensure that the participants did not deviate (Mahlomaholo, 2009). The observation sessions gave the researcher an opportunity to observe the teaching practices (e.g. how the UDI principles were adapted to teach geometry). The focus group discussions gave the participants opportunities to reflect on their practices, thus explaining how they adapted the UDI principles in their classrooms to provide flexible, inclusive and accessible teaching.

The study was ethically cleared by the University and permission to conduct the study was granted by the Department of Education (DoE) and the principal of the school where the study was conducted. The participants (teachers) signed consent forms to participate in the study and for the lessons and discussions to be recorded. The parents also signed the assent forms to give consent for the lessons to be video recorded in classes.

Content analysis methods were used to analyse data. The content analysis methods were directed because the general themes were determined *a priori* (McNamara, 2018). This means that the principles that were used as themes (in this context) were predetermined as they were taken from the UDI framework as they were. However, the teachers in the school where the study was conducted applied them according to their understanding and interpretation, which probably could have been different if the implementation of the principles was carried out in a different context (another school). On the basis of this, the results of this study cannot be generalised. However, the findings can contribute to different contexts and to the body of knowledge in general by highlighting some UDI-inspired practices that could be adapted to promote flexible, inclusive and accessible teaching.

Findings and discussion

The following sections provide the findings and discussions based on the data generated through focus group discussions and lesson observations.

Focus group discussions

The data reported in this section were generated through focus group discussions. The participants narrated and reflected on how they implemented the UDI principles namely, equitable use, low physical effort and instructional climate.

Equitable use

The equitable use principle of UDI (Appendix 1) requires the instruction to be designed in a manner that is useful to and accessible to all learners (Burgstahler, 2009). This means that the instruction should provide the same means of use for all learners, identical whenever possible and equivalent when not. To provide the teaching of geometry that is supportive of this principle, this study's participants reflected as follows.

> Learners are different. For example, when you explain the properties of the triangles, there are those who will catch what you are saying quickly by listening to your explanations, whilst there are those who will understand when you are explaining and showing them the properties at the same time. Therefore, when you know all this about your learners it becomes easier to use the appropriate methods of teaching to cater for the different categories of learners in your class. (Teacher 4)

> Let me show you what equity means [projecting Figure 1 on the screen]… you see this picture shows the principle that we should be applying in our classrooms when we teach. We have different learners and therefore we need to make sure that we strive to put them on equal level as shown in this picture. So how do we do that? ... That is an important question we need to ask ourselves. We have to formulate the teaching strategies that will enable all learners to learn, whilst taking into consideration their different learning styles and preferences in order not to cause segregations by using certain teaching strategies that favour certain individuals. (Teacher 2)

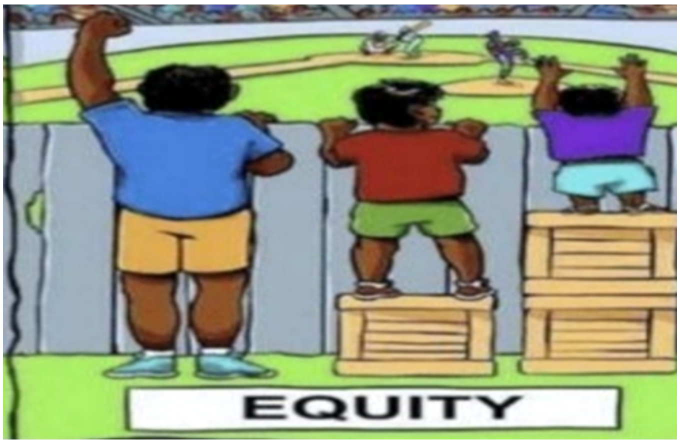


Figure 1: Equity principle (Teacher 2)

The participants stressed the need to use equitable teaching strategies as part of implementing "Equitable use". According to Teacher 4, the fact that learners differ in terms of how they learn, requires teachers to devise teaching strategies that are suitable to their individual learning styles and preferences; hence the statement "*use the appropriate methods of teaching to cater for the categories of learners in your class*". Teacher 4, however, indicated that teachers need to know the different characteristics of their learners to be able to devise such "appropriate" strategies. Teacher 4 thus espoused the concept of differentiated instruction (which is aligned with theories such as Vygotsky's *zone of proximal development* (Vygotsky, 1978) and Gardner's theory of *multiple intelligences* (Gardner, 1993), which is seen as a philosophy and a praxis of teaching emerging from reform efforts that are aimed at individualising teaching, in order to achieve equity in the classrooms (Valiandes, Neophytou & Hajisoteriou, 2018).

Teacher 2 agreed with the notion of devising equitable teaching strategies that give all learners access to learning, regardless of their different characteristics (e.g. learning styles and preferences). The analogy that Teacher 2 used (Figure 1) signifies the importance of striving to put learners at the same level of accessing and understanding geometry concepts by providing the same means of use for all learners; identical whenever possible and equivalent when not. This assists in avoiding segregation that may arise as a result of teachers being inclined to use practices that are only suitable or appealing to a particular group of learners, instead of all learners in the classrooms.

Low physical effort

The low physical effort principle promotes instruction that is designed to minimise non-essential physical effort in order to allow maximum attention to learning (Burgstahler, 2009). This principle and its application were highlighted during the FGDs. Participants highlighted as follows.

> Often as teachers we complain about time, saying we are unable to use certain strategies in our classrooms because they take time to implement. But that is not always true because I have seen that when you prepare thoroughly in advance, things just run smoothly in class and you actually start realising that you save time instead! Coming to class with a *Geo-Trig* board (Figure 2) has helped me save time and energy. Instead of drawing on the chalkboard, I use it with the rubber bands to teach and demonstrate different geometrical shapes. I like it because its measurements are accurate. (Teacher 5)

> Remember, the aim is to engage the learners in the process, so it also helps to come with the ready-made materials and give them to the learners. Instead of their drawing, taking too much time to do so, they can just use the materials as instructed and this also saves time and energy. (Teacher 7)

> It helps to use the ready-made materials such as *Geo-Trig* boards because they are more flexible. For example, instead of focusing on the geometric figures provided in the textbook or redrawing them on the board as we always do, we could use these materials. You see, what I like about these materials is that you can play around with them to change their different positions and that will help learners to be able to recognise the shapes even when they are faced in different directions or mixed. This helps because often learners recognise the shapes by the manner in which they are often represented, without thinking of other ways in which the same shapes could be represented". (Teacher 4)

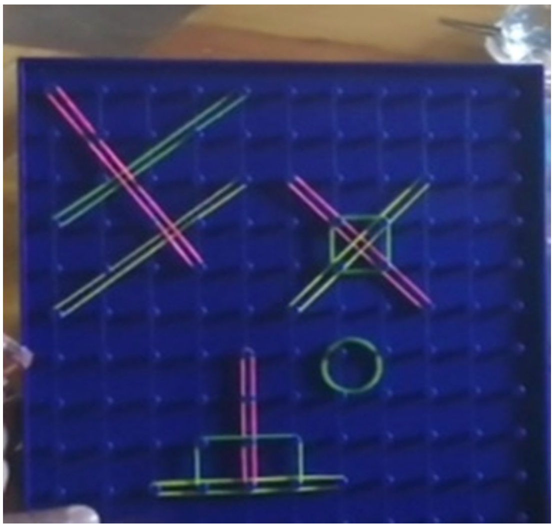


Figure 2: Geo-Trig board (Teachers 5 and 7)
[A board that is used to demonstrate geometry and trigonometry shapes and concepts]

Teacher 5 criticised the claim that teachers often say that they are unable to use certain strategies in their classrooms because the strategies take time to implement. The teacher said that is not always true because when thorough preparation is done in advance, teaching becomes efficient; hence the statement *"when you prepare thoroughly in advance, things just run smoothly in class and you actually start realising that you save time instead!"*. In line with this, Teacher 5 highlighted how the *Geo-Trig* board has helped him save time and energy. Thus, Teacher 5 used this instrument to replace the work that he would normally be expected to do in terms of drawing the geometric shapes on the chalkboard. The instrument is therefore useful in terms of demonstration purposes and also effective since its measurements are accurate. Teacher 7 shared the same sentiment and also endorsed the use of "ready-made" materials which should be given to the learners, instead of engaging them in the strenuous labour of drawing the geometric shapes, whilst also expecting them to learn the geometric concepts simultaneously. In line with the UDI principle of low physical effort, teachers thus designed an instruction, which minimised the non-essential physical effort in order to allow maximum attention to learning.

According to Teacher 4, the *Geo-Trig* board can be used to illustrate the different orientations of the geometric shapes and this can be useful in terms of enabling learners to recognise the shapes and their properties, regardless of the different positioning. Teacher 4 emphasised that often learners tend to recognise the shapes by the manner in which they are often represented and they do not think of other ways in which the same shapes could be represented. Thus, playing around with shapes to show their different orientations enables learners to perceive or recognise the shapes regardless of their positioning or being mixed as shown in Figure 3. The use of "ready-made" materials (*Geo-Trig* board) in this manner, epitomises the "flexibility in use" principle, in which learners are engaged in varied teaching methods (Scott et al., 2003).

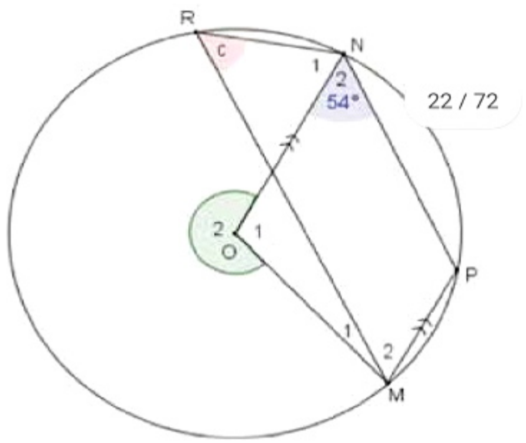


Figure 3: Complex geometric figure (Teacher 4)

Instructional climate

The instructional climate principle promotes instruction that is designed to be *welcoming and inclusive*, wherein high expectations are espoused for all learners (Scott et al., 2003). This UDI principle, as well as its application, was also highlighted during the FGDs as useful in terms of promoting learning. Participants' responses are highlighted as follows.

> I try to always explain to my learners what is expected of them in terms of the concepts to be learnt, what they are expected to do in order to do well, and how assessments will be done. I find this helpful in terms of making them aware of my expectations for them and also what is expected of them when learning geometry. (Teacher 8)

> Giving them equal opportunities to learn, opportunities to try to solve problems on their own, to allow them to commit errors without being judged, and to learn from their mistakes make learners feel they belong. (Teacher 3)

> Making learners realise the importance of learning geometry and relating it to their life experiences helps in terms of motivating them. We need to realise that even if we plan our lessons very well, if we are not going to create a warm and an enabling environment, our learners will still struggle to learn. (Teacher 2)

Teachers highlighted the need to create an instructional climate that is enabling and welcoming to promote learning. Teacher 8 highlighted the significance of designing an instruction in a manner that is welcoming and inclusive. She indicated that in order to do so, she always explains to learners what is expected of them in terms of the concepts to be learnt; explains what they are expected to do in order to pass; and how the assessments will be done, thus communicating the expectations to all learners. Teacher 8 indicated that she sets expectations to all learners in terms of making them aware of what is expected of them in learning geometry. Promoting a sense of belonging for learners was also deemed significant by Teacher 3, who cited the idea of giving learners equal opportunities to learn, equal opportunities to try to solve problems on their own, and giving them the latitude to commit errors without feeling judged. In addition, learning from their mistakes at the same time, is an important strategy to uphold.

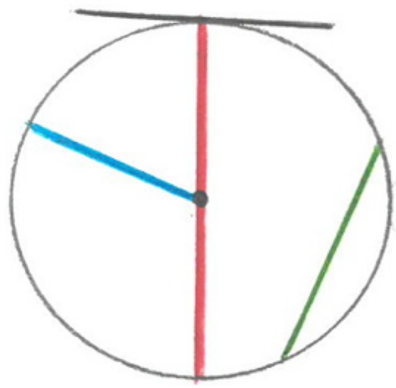
Another aspect mentioned by Teacher 2 was the need to make learners aware of the significance of learning geometry by relating it to their life experiences. This promotes, the notion of situated learning, which requires learning to be embedded in authentic contexts of practice, wherein learners engage in tasks within social communities (Cook & Yanow, 1993). Teacher 3 further pointed out that:

> Even if we plan our lessons very well, if we are not going to create a warm and enabling environment, our learners will still struggle to learn. (Teacher 3)

Lesson observations

The data reported in the subsequent sections were generated through lesson observations. The sections indicate how the following UDI principles: *perceptible information; simple and intuitive; flexibility in use; a community of learners; tolerance for error, and low physical effort* were implemented.

Observation lesson extract: Teacher 1 activity done in class with the learners

<table>
<tr><td>Descript-
ion</td><td colspan="2">Teacher 1 taught learners about the geometry of the circle and during the lesson, highlighted some of the important features of that particular circle (i.e. different types of lines). The session was hands-on wherein the teacher engaged the learners in the learning process. The class comprised of 29 learners who were divided into 7 groups of 4. One of the groups comprised of 5 learners since the total number of learners was 29. Two big tables and 4 single chairs were allocated per group so to enable 2 pairs of learners to face each other in the group. One group was allocated an extra fifth chair since the number of learners was 5 in that group.
The dialogue below outlines what transpired during the lesson.</td></tr>
<tr><td>Teacher 1</td><td colspan="2">[giving instructions] Draw the circle and a point at the centre of that circle.
Using a red colour draw a straight line from the circumference through to the point at the centre to the other point on the circumference.
Using a blue colour draw another line from the centre of the circle to the circumference.
Use a black colour to draw another straight line that touches a curve at a point, but if extended does not cross it at that point.</td></tr>
<tr><td>Learner 3</td><td colspan="2">[asking a question] Teacher, what is a curve?</td></tr>
<tr><td>Teacher 1</td><td colspan="2">Good question… you see I regard a circle as a curve. To me, a circle is a simple closed curve.</td></tr>
<tr><td>Learner 3</td><td>Okay teacher!</td><td rowspan="5">
Figure 4: Circle with different lines</td></tr>
<tr><td>Teacher 1</td><td>All right then! … Draw a straight line that connects two points on a curve using a green colour.</td></tr>
<tr><td>Teacher 1</td><td>Now that you have followed all my instructions you should have at least a drawing like this one [showing the drawing on paper (Figure 4)]. Do you have something like this?</td></tr>
<tr><td>Learners</td><td>[learners responded at the same time, some saying yes, others saying no]</td></tr>
<tr><td>Teacher 1</td><td>Okay, let us proceed … the red line is called diameter, the blue line is called a radius … we call the green line a chord and a black line a tangent….</td></tr>
</table>

In the interactive lesson above, Teacher 1 engaged the learners in a lesson wherein the aim was to teach them about the different types of straight lines used in the geometry of a circle and to help them distinguish the lines. Instead of using the "telling method", which is the traditional style of teaching which regards learners as passive recipients of information (Kaur, 2011), Teacher 1 engaged them in the actual drawing of the circle and the lines using different colours to be able to separate them. The use of the different colours assisted in terms of enhancing visual perception, thus making information perceptible, simple and intuitive to all learners. Consequently, learners were able to see how these lines looked individually, as well as how they differed. The act of engaging learners in the actual drawing served as a good strategy to enable them to participate and understand informal deductive discussions about the different types of straight lines presented and their different characteristics.

Observation lesson extract: Teacher 3 activity - features of a parallelogram

Teacher 3	*[holding a paper]* A line that is joining two opposite angles, we call a diagonal … and I am sure you can already see a "zet", alternating angles! (referring to the picture projected on the screen (Figure 5) ... Hare re motho o dula opposite le ntlo ya hao re reng? *[English translation: If we say a person stays opposite your house, what do we mean?]*
Learner 9	It means dintlo tsa rona di shebane *[English translation: It means our houses are facing each other]*
Teacher 3	Correct … you see here *[pointing to the two opposite angles – see* Figure 5*]*, these two angles are facing each other. They are opposite to each other. All right … now diagonal … *"di"* – means two … two corners [referring to Figure 5]. Oooops! In maths we refer to the corners as angles! Please make sure that you observe the symbols that I am using. These symbols are important because they give you more information.  Figure 5: Picture of a diagonal line joined by two opposite angles

Using paper as a demonstration instrument, Teacher 3 explained some of the properties of a parallelogram, thus operationalising the UDI principle of "simple and intuitive". Teacher 3 designed an instruction in a straightforward, simple manner. To enable learners to understand the meaning of "opposite angles", Teacher 3 first asked a question related to the use of the term opposite ordinarily (i.e. "if we say a person stays opposite your house, what do we mean?"). Teacher 3 asked the question intentionally, since he knew that the meaning of the term when used ordinarily, would still be applicable in the context of what he was teaching and also assist all the learners to identify the angles that were positioned in that way. This strategy did not only make the concept simple and intuitive but also, it was useful in terms of eliminating the unnecessary learning complexities.

A deduction that was made during the lesson by the researcher is that making a lesson simple and intuitive requires not only the design of a simple straightforward lesson but also the explanations of the mathematical terms that are used in geometrical expressions; hence the explanation *"now diagonal… "di" means two…"*. It also requires the teachers to be mindful of the "informal" language that they are using for clarity / simplification purposes and to be able to correct that "informal" language by putting it in context; for example; *"In maths we refer to the corners as the angles!"*. According to Teacher 3, making the concept simple, intuitive and perceptible requires the teaching of symbols that are used and incorporated into shapes (Figure 6). In addition, teachers need to ensure that learners observe symbols, since they are useful in terms of providing more information on geometric shapes. To make the concept simple, intuitive and perceptible, Teacher 3 explained the diagonal line in relation to the formed letter "z" and the types of angles a diagonal line forms namely, alternating angles (as shown in Figure 6).

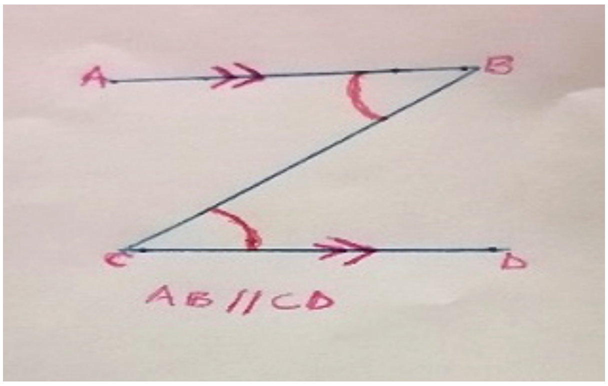


Figure 6: Diagonal line drawn between two parallel lines forming alternating angles

Observation lesson extract: Teacher 6 activity - properties of triangles

Teacher 6	Let us look at the picture on the screen (*see* Figure 7). Have you seen such a shape before? If you have, which shape is that?
Learner 2	It is a kite Teacher!
Teacher 6	Good ... Now, let us look at its features! I want you to discuss in your groups which triangles or what types of triangles we have there and report back.
Learner 1	*[from group 2]:* We have the right angle triangle and the isosceles triangle.
Teacher 6	How do you know that one is a right angle triangle and the other an isosceles?
Learner 1	Because one has an angle of ninety degrees and the other one has two sides which are equal.
Teacher 6	Mmhhh interesting! Now let us look at picture 2 (*see* Figure 8) How is it different from picture number 1?
Learner 7	*[from group 4]*: The second picture is marked and those marks are the ones that indicate what type of triangles we have.
Teacher 6	Good! You see, you cannot just assign names to triangles without considering the signs.

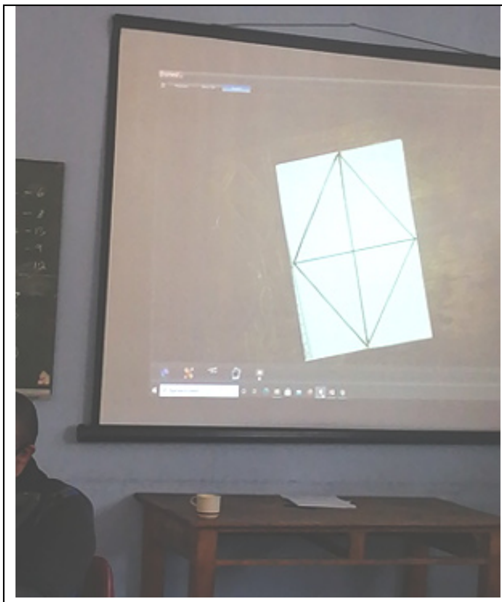
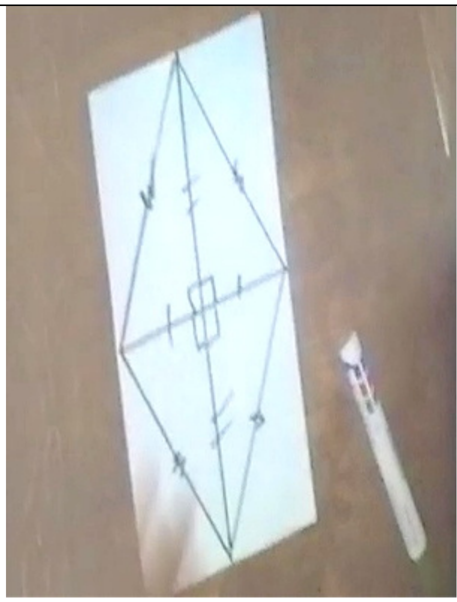
	

Figure 7: Kite	Figure 8: Kite with labels

Teacher 6 wanted learners to discuss and identify in their groups the triangles that were represented in the shape (kite) (as shown in Figures 7 and 8). Learner 1 (who belonged to group 2) indicated that they are right angled and isosceles triangles. The teacher realised though that the learners in that group recognised the shapes by their whole appearance, but not their exact properties. To address the "mistake", she gave a similar shape, but this time with "labels", as shown in Figure 8.

In response to the second question, Teacher 6 asked "Now let us look at picture 2 (referring to Figure 8), how is it different from picture number 1 (i.e. Figure 7)?" Learner 7 (who represented group 4), pointed out that the second figure (i.e. Figure 8) had "marks" on it that make it possible to recognise and indicate the types of triangles depicted therein. Teacher 6 then used Learner 7's correct response to emphasise the importance of the "marks" that were incorporated in the shapes, as shown in Figure 8. Teacher 6 emphasised that names cannot simply be given to the triangles without consideration of the "labelling marks", which is a mistake that was done by Learner 1. The labelling of marks is important as it provides descriptions and information necessary to understand, reason and solve problems. The reality that Teacher 6 did not just give learners answers but provided them with opportunities to first think on their own and come up with answers (i.e. those that were correct and incorrect), denotes the application of a UDI principle known as "tolerance for error". Teacher 6's act of emphasising the importance of "labelling marks" was intended to correct the learners' tendency to give names to shapes based on what they think of shapes in terms of what they look like.

In the above lesson, Teacher 6 provided learners with an opportunity to work in groups, a strategy that provides learners with opportunities to share ideas and to co-construct knowledge in the learning process (Laal & Laal, 2012). In line with this, Teacher 6 used a

UDI principle called "community of learners" which promotes interaction and communication among learners and between learners and teachers, as shown in the sociogram in Figure 9.

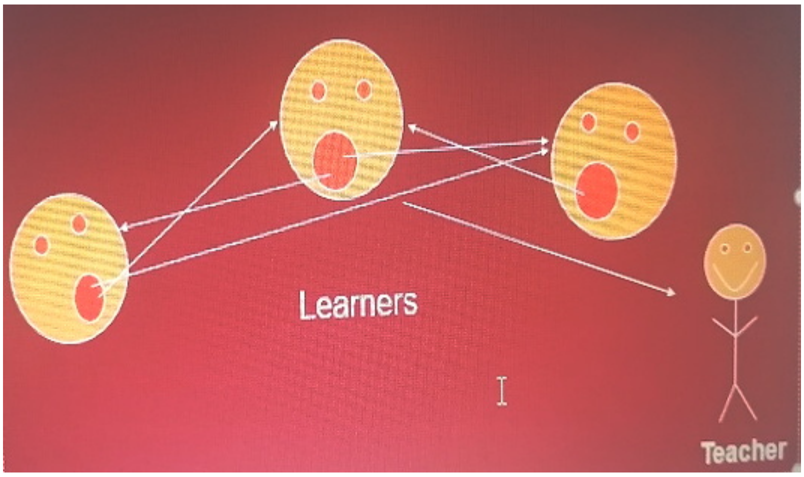


Figure 9: Sociogram

Conclusion

This study addresses an important issue in mathematics namely, flexible, accessible and inclusive teaching of geometry to maximise learning. This study has revealed that teaching which is not flexible, accessible and inclusive can impede learning and potentially lead to feelings of exclusion, particularly in diverse groups of learners for whom predominantly used traditional methods of teaching do not respond to their learning needs. Findings also revealed that methods of teaching which are not flexible, accessible and inclusive deprive learners of opportunities to engage in meaningful learning. This study thus provides a UDI framework to serve as a guide for devising flexible, accessible and inclusive geometry teaching strategies to maximise learning. The study indicates that teachers need to use equitable and flexible strategies. Teachers also have to use strategies that make geometry concepts simple, intuitive and perceptible. Teachers also have to give learners opportunities to commit errors and to learn from such errors. The use of the ready-made materials was found to be useful in terms of reducing physical effort while allowing maximum attention to learning. Providing learners with opportunities to work collaboratively was found to be useful in terms of maximising learning.

On the other hand, devising equitable and flexible teaching strategies was found to be beneficial for a broad range of learners and subsequently served as a good strategy for creating a conducive instructional climate that is necessary to promote and maximise learning. Although the principles are highlighted and defined separately in the study as shown in Appendix 1, when they are applied in class, this study shows that they overlap. This means that when one principle is applied, its application epitomises the application of the other principle at the same time. For instance, flexible methods of teaching can make geometry concepts simple and intuitive, as well as perceptible. In an endeavour to provide teaching that is flexible, accessible and inclusive to maximise the learning of geometry, the study shows how UDI principles can be applied and, as such, suggests the need for teachers to consider these principles when planning geometry lessons, as well as when teaching geometry. The study thus shows UDI as a potential strategy that could be explored further in terms of maximising the learning of geometry and other topics in mathematics.

As already alluded to at the beginning of the article, the implications of this study are positioned within the debates on promoting inclusive education to enhance the teaching and learning of mathematics, as underscored in the results of some international benchmark assessments, such as *Trends in International Mathematics and Science Study* (TIMSS). In line with this, therefore, the findings of the study suggest the need for efforts to be made to teach geometry in a manner that is flexible, accessible, and inclusive. This is the approach that teachers should follow to teach geometry and mathematics in general, for understanding and to accommodate all students in the teaching and learning process. Approaching the teaching of geometry using UDI principles therefore requires teachers to plan lessons carefully in a manner that promotes learning accessibility. This approach also serves as a good way to assist in informing better teaching and assessment of student learning; thus, overall promoting quality teaching.

Limitations of the study

One of the limitations of this study regarding UDI is the absence of "ICT integration". The study did not highlight ways in which ICT can be integrated into the teaching of geometry and also how UDI principles could be used to guide flexible, accessible and inclusive online teaching. The second limitation is that the study was conducted in one school and therefore its results cannot be generalised. However, the results of the study can offer great lessons for guiding flexible, accessible and inclusive teaching and learning in mathematics and other subjects.

Priorities for future research

The integration of ICT into the teaching of geometry guided by UDI principles should be further investigated to promote flexible, accessible and inclusive online teaching and learning. Furthermore, comparative studies involving rural, farm and urban schools should be conducted to determine how the different contexts can inform UDI implementation.

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Appendix 1: UDI principles and applications in teaching

	Principle	Application to teaching
1	Equitable use	Instruction is designed to be suitable for and accessible to learners with diverse abilities. ▪ Provide the same means of use for all students; identical whenever possible, equivalent when not.
2	Flexibility in use	Instruction is designed to accommodate a wide range of individual abilities. ▪ Provide choice in methods used.
3	Simple and intuitive	Instruction is designed in a straightforward and predictable manner, regardless of the student's experience, knowledge, language skills, or current concentration level. ▪ Eliminate unnecessary complexity.
4	Perceptible information	Instruction is designed so that the necessary information is communicated effectively to the student, regardless of ambient conditions or the student's sensory abilities.

5	Tolerance for error	Instruction anticipates variations in individual student learning pace and prerequisite skills.
6	Low physical effort	Instruction is designed to minimise nonessential physical effort in order to allow maximum attention to learning. Please note: This principle does not apply when physical effort is integral to essential requirements of a course.
7	Size and space for approach and use	Instruction is designed with consideration for appropriate size and space for approach, reach, manipulations and use, regardless of a student's body size, posture, mobility, and communication needs.
8	A community of learners	The instructional environment promotes interaction and communication among students and between students and faculty.
9	Instructional climate	Instruction is designed to be welcoming and inclusive. ▪ High expectations are espoused for all students.

(after Scott et al., 2003, pp. 375-376)

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