'Does mathematics fool us?' A study on fourth grade students' non-routine maths problem solving skills

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This study used a holistic multi-case research design to investigate fourth-grade students' skills for solving non-routine mathematics problems. The participants were three 4th grade students (primary school) with low, moderate and high achievement in mathematics respectively, who were educated in the same class at a state school of middle socio-economic level in north-east Turkey. They were selected through a maximum diversity sampling method. Data were collected using a clinical interview technique, via a form consisting of four non-routine problems, developed by the researcher, and were analysed via a content analysis procedure. Findings reveal that participants did not use strategies that can be used during reading problems; although all three students performed the arithmetic operations correctly, they could not reach the correct result in their first stage in both problems; they only checked the accuracy of the result on arithmetic operations.

Introduction

Primary schooling is the period when students acquire basic skills, some of which are closely related to mathematics. Mathematics is an essential subject that should be learnt well in primary school due to its contribution to the development of individuals' multi-faceted and relational thinking skills as well as problem-solving skills.

Problem solving

Mathematics and problem solving are two closely intertwined phenomena. In that regard, Dossey (2017) advocated that problem-solving constitutes the heart of mathematics. It is sometimes regarded as a method for teaching mathematics and the goal of learning mathematics (Liljedahl, Santos-Trigo, Malaspina & Bruder, 2016). These two aspects of problem-solving reveal its importance as a skill. Problem-solving is a dynamic and flexible process since problem analysis and solution construction are intertwined and interacting processes (Sprenger, 2007). Problem-solving comprises at least four stages: examination and understanding; establishing relationships and formulating; planning and execution; and monitoring and reflection (OECD, 2013). Problem-solving is not solving questions, but solving concerns. Problems often involve complex and multifaceted solutions that contrast with questions having pre-determined solutions and unique correct responses (Olkun & Toluk-Uçar, 2014). Therefore, memorised and mechanical knowledge does not often work in problem-solving. Van de Walle, Karp and Bay-Williams (2014) postulated that students who are accustomed to being told how to do mathematics do not attempt to solve the problem unless explicit instructions are provided on how to solve it. Doing mathematics includes finding ways to solve problems.

Problem-solving skills are identified among the target skills at all grade levels in primary mathematics curricula, as the students who acquire these skills successfully are more likely to overcome daily problems with ease. However, these problems are not standardised; some of them may not be solved if experience has been gained only through routine learning processes. For that very reason, primary school students are expected to be engaged both with the problems that require routine processes and those that require relational, multifaceted and creative thinking skills compatible with their cognition. In addition, good problems are identified as those which are related to students' environment, which forces them to develop and implement strategies and enable them to acquire new concepts (NCTM, 2000).

Isık-Tertemiz, Doğan and Karataş (2017) held that students tend to recall specific rules when encountering a problem; however, it is not an appropriate way because problem-solving requires systematicity rather than rules. A successful problem solver should have an in-depth understanding of mathematical concepts, specialisation in problem-solving strategies and techniques, a positive attitude and belief towards mathematics and problem solving, and the ability to make right decisions (Schoenfeld, 1985; Doğan & Çetin, 2018). Mayer (2004) informed that problem solving involves going through the following cognitive processes:

1. Mental rearrangement of each sentence;
2. Integrating knowledge to form a mental representation of the whole problem;
3. To plan a solution and follow this plan in the process of problem-solving;
4. Perform the solution procedure.

In the world of mathematics, problem solvers can transform real-life problems into a more readily comprehensible format (Mataka, Cober, Grunert, Mutambuki & Akom, 2014). This is possible, by and large, with performing the first two stages of Mayer's (2004) processes.

Non-routine problems

Pólya (1966) stated that problems are classified into "routine" and "non-routine" problems and that the latter require students' creativity and originality. Routine problems are mostly in the form of exercises. It may be difficult to distinguish between an exercise and a non-routine problem, due to the fact that a task is perceived as an exercise by some students, whilst others might perceive it as a non-routine problem based on their knowledge, experience, and equipment (Carlson & Bloom, 2005; Zhu & Fan, 2006). Schoenfeld (1992) stated that routine problems are exercises and that non-routine problems differ from exercises in that they are surprising and requiring non-standard algorithmic strategies. For example, the question, "How many passengers can a bus, which can carry up to 25 passengers, transport in 3 trips?" is a routine problem for many students because they do not need higher-level thinking skills to solve it. They just need to possess the knowledge of addition or multiplication. The problem can be simply solved by the operations of 25x3 = 75 or 25+25+25 = 75. However, the question of "How many trips should be conducted by a bus, which can carry up to 25 passengers per trip, to transport 59 passengers?" is different from the first version. It is a non-routine problem for many students as they need to be careful and to think more analytically by avoiding the misleading response of "2" that is obtained from the operation of "59 ÷ 25 = 2". It is necessary to consider that the bus should carry as many passengers as possible to transport them in as few trips as possible, and that a third trip is needed to transport the last 9 passengers.

Non-routine problems have multiple solutions and solutions may differ among students (Robinson, 2016). Non-routine problems require attention, analytical thinking, and creativity and more cognitive effort than routine problems (Mullis, Martin, Ruddock, O'Sullivan & Preuschoff, 2009). This effort might also be explained by the concept of "accommodation" in Piaget's (1936) cognitive theory. Students may engage in cognitive regulation to understand and produce solutions to non-routine problems that they failed to solve with familiar strategies. This effort may contribute to new learning and facilitating the development of their multi-faceted thinking skills.

Non-routine problems could also form the basis of the STEM applications (science, technology, engineering, and mathematics), which have emerged as a requirement in this era. The use of mathematics in STEM applications overlaps with the nature of non-routine problems, which have features such as proximity to real life, multidimensional solvability, and the need for functional and relational thinking that appear among the essentials of STEM applications.

Dealing with non-routine problems can provide the following benefits to students (Lampert, 2001: 3):

- Increase motivation and performance towards learning;
- Increase interest in school and enhances the understanding;
- Increase strategic thinking capacity;
- Improve ability to link thoughts;
- Make teachers more accessible for a wider learner population;
- Prepare students for the business world.

Due to the aforementioned reasons, it would be appropriate to include non-routine problems into maths curricula and to allocate time for these problems in mathematics classes. However, Lee and Kim (2005) reported that teachers infrequently use non-routine problems in their teaching. Likewise, Asman and Markovits (2008) reported that teachers could include non-routine problems in their classes, but they would not ask such problems in exams, so that the students could pass the exams. However, Yazgan's study (2007) showed that fourth and fifth-grade students developed different strategies for non-routine problems and could think in a multidimensional manner. Ulu, Tertemiz and Peker (2016a) concluded that training fifth-grade students in reading-comprehension strategies increased their success in solving non-routine problems. These studies indicate that students can be successful in solving non-routine problems. However, Artut and Tarım (2006) concluded that fifth-grade, seventh-grade and eighth-grade students failed to solve non-routine problems. Chacko (2004) indicated that both primary and secondary school students solve non-routine problems as if they were routine problems. Within both studies, this situation was attributed to the absence of non-routine problems in mathematics curricula, and students being accustomed to routine problems.

This study aimed to make an in-depth examination of fourth-grade students' skills to solve non-routine mathematics problems and to give suggestions based on the findings.

Method

Research design

This qualitative research was designed as a case study since it aimed to make a detailed description and analysis of a limited number of cases. Case studies are a form of research in which one or more current situations are described and analysed in detail, how and why questions are sought to be answered, and the control area of the researcher is limited (Christensen, Johnson & Turner, 2015; Yin, 2018). They aim to discover, describe, and interpret cases or people in their original environment rather than draw general conclusions (Parker, 2015). This study is a holistic multi-case study conducted with the participation of a few individuals. Such case studies are motivated to focus on detailed results about a limited number of individuals and to identify determinants, factors and processes that may affect the outcome (Hakim, 2000). In addition, cases are compared after individual analysis (Yıldırım & Şimşek, 2008).

Participants

The participants were three 4th grade students (two female and a male) with low, medium and high achievement in mathematics who were educated in the same class in the spring semester of the 2018 year, at a state school of middle socio-economic level in north-east Turkey. They were chosen through a maximum diversity sampling method, which requires identifying different situations related to a particular problem and grounding the research on these situations (Büyüköztürk, Çakmak, Akgün, Karadeniz & Demirel, 2012). Their mathematics achievement in the first three years of their primary schooling, as well as their classroom teachers' opinion about them, were taken into consideration in sample selection. Based on the "*Regulation on Pre-school Education and Primary Education Institutions of the Ministry of National Education*", their achievements in 1st, 2nd and 3rd grades were stated as "*Very good, Good* and *Developed*", respectively. Their end-of-term success in mathematics in the first semester of 4th grade was taken into consideration together with their achievement in the first three grades (Table 1).

Table 1: Information about participants

Pseudonyms	Age, (gender)	End-of-term grade (maths)	Achievement in grades 1-3 (maths)	Achievement level in maths
Safiye	10 (F)	90	Very good	High
Burak	10 (M)	60	Good	Medium
Özlem	10 (F)	35	Developed	Low

Data collection

Data were collected through a form consisting of four non-routine problems developed by the researcher. Expert opinion was elicited from three academics with 8-10 years of experience and five classroom teachers with 5-17 years of experience on the compatibility between problems and learning outcomes, clarity and item compliance. As a result, two problems were removed from the form due to their low compatibility with learning outcomes. The remaining two problems were piloted to 12 non-participant students (4 low, 4 medium, and 4 high achievements in maths). The form was finalised after the evaluation of their responses by the Researcher and a classroom teacher, for errors caused by the problems, and their conformity to the student level. In addition, clinical interviews were conducted with 2 non-participant students to identify any problems that might occur during the interviews.

The following are the non-routine problems used in the collection of research data (translations from Turkish to English here and elsewhere are by the author):

Problem 1: The length of a circular bike trail is 400 metres. One has to pay 10 TL to take a bike tour in the trail at the beginning of each tour. How much should Salih pay for riding 600 m on this trail?

Problem 2: The bus stops are located at 500 metre intervals in the province of Bayburt. Feyza got on the bus at the 1st stop and got off at 6th stop to go to the school. In this case, how many metres did Feyza go to school by bus?

Clinical interviews

The clinical interview is a data collection technique that provides an in-depth examination of the situation by posing further questions to the individuals in order to gain a better insight into their intellectual wealth (Baki, Karataş & Güven, 2002). Ginsburg (1981) argued that clinical interviews are appropriate for examining students' mathematical thinking, exploring their cognitive processes, and evaluating their skills.

Prior to data collection, pre-interview sessions were held with each student to motivate them to solve the problems and to decrease their anxiety. To do this, the Researcher told them "*I have a difficult problem to solve. I would appreciate if you could help me solve it*". Each student was interviewed at different sessions to prevent modelling error. Each student attempted to solve only one problem in each session. A second session was held with the same student for the other question two weeks later. The sessions were held in the Deputy Director's room outside of class hours. Durations for Safiye were 26 and 31 minutes; for Burak 20 and 21 minutes; and for Özlem 18 and 14 minutes.

The students' classroom teacher was also available during the sessions and took observation notes. These notes were also used to report the research findings. The report was shared with the classroom teacher to elicit his/ her opinion. All in all, the construct validity and reliability of the research were tried to be increased through the above-mentioned processes. The sessions were audio-recorded with the prior consent of the students and their teacher. Based on the clinical interview technique the recorded data, students' answer sheets, and the Researcher's and classroom teacher's observation notes were analysed through content analysis.

Findings

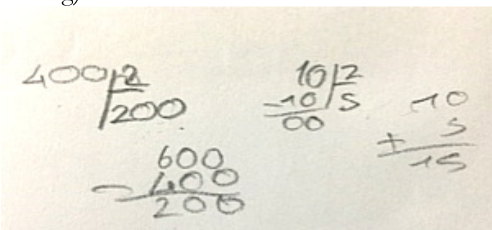
In this section, the findings obtained from clinical interviews with each student are presented.

Safiye's problem 1 solving process: Bike trail

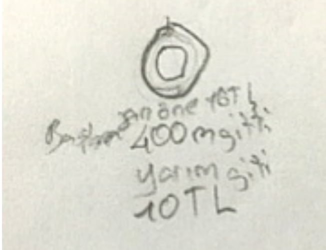
Safiye's solution for Problem 1 and the interview with her are given in Figure 1. Safiye read the above problem silently. She did not use strategies such as highlighting important points during the reading, noting those given, circling and drawing figures. However, she was able to express the problem in her own sentences. According to Safiye's verbal expressions, it can be claimed that she did not have any difficulty understanding the problem. Safiye was able to make quick reasoning and then reached the conclusion as *15 (fifteen)* by performing mathematical operations from her mind without using paper and pencil. Safiye was asked to illustrate those operations on the paper, and it was observed that she performed the operations arithmetically correct. She checked the accuracy of the conclusion only through arithmetic operations rather than making any relational control over the data. Safiye attempted to reach the conclusion through a simple proportion but failed even though she made correct arithmetic operations. This finding shows that Safiye did not read the problem carefully or she could not determine the appropriate strategy to solve the problem as she routinely thought about organising what she reads, classifying and identifying the relationships between the given data.

When asked to solve the problem by drawing a figure, Safiye was able to position the ones given in the problem on the figure, but she did not initially realise that full tour money should also be paid for half a tour. Once the researcher put the emphasis on the statement "*A full tour fee is paid at the beginning of each tour*" in the problem and repeated the question, Safiye took the pencil on the track image drawn and gave the answer "*1.5 laps*". After Safiye remained silent for a while, the Researcher asked the question "*Does one have to pay for a half-round?*" Subsequently, Safiye, once again, moved her pen over the track image she drew and stopped it once she completed a full trail on the image. Thanks to the image she drew and the researcher' question she recognised that one has to pay for another full trail to continue after the finish line (even though she needs to ride for a half trail) and thereby, she obtained the correct result. According to this finding, it can be claimed that the use of the drawing strategy makes it easier for Safiye to solve the non-routine problem and reach the correct result.

Researcher: What did you understand about the problem?
Safiye: There are 400 metres bike trail. One has to pay 10 TL for the trail, and if he proceeds 600 metres, how much TL does he have to pay?
R: What can we do in this situation?
Safiye: In this case, he drives 400 metres first, and paid 10 TL. But how much does he have to pay for a ride of 600 metres? I'll find it.
R: How will we find it?
Safiye: He paid 10 TL. He has to ride 200 more metres. In other words, half of 400 is 200 metres, half of 10 TL is 5 TL. Then, it costs 15 TL.
R: Are you sure about your answer?
Safiye: Yes! Half of 400 is 200; half of 10 is 5.
R: So, can you show me how you found it (by writing)?



R: Why did you divide 400 by 2?
Safiye: Because I will find half of it. Because he ran halfway!
Safiye: When I subtract 400 from 600, it becomes 200. Half of 400 is 200! He ran halfway! If we divide 10 by 2, he has to pay 5 TL. He had paid 10 TL! If we add them, he has to pay 15 TL.
R: Are you sure about your answer? Do you want to check?
Safiye: Hmm. He had to pay 10 TL before he started. No. I think that's the answer (15 TL).
R: Do you want to solve the problem by drawing?
Safiye: Sure.

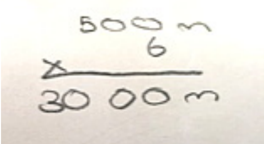


R: Can you tell me your solution through the shape?
Safiye: He went 400 metres and paid 10 TL. If he goes 200 more metres, he will pay 5 TL more; it becomes 15 TL.
R: So what does the expression "A full tour fee is paid at the beginning of each tour" mean?
Safiye: Hmm. Full tour.
Rr: How many times does Salih have to tour around to ride 600 metres?
Safiye: He tours around once. He needs a half more round.
R: So how many rounds does he tour?
Safiye: 2.
R: 2 rounds or 1.5 rounds?
Safiye: 1.5 rounds.
R: And does one have to pay for a half-round?
Safiye: Yes, he does.
R: How did you understand that?
Safiye: Because he pays 10 TL at the beginning of each round… This means….! It's 20.
R: How?
Safiye: He finished one of them and came again! He paid 10 TL before he started. He paid 10 TL more; it becomes 20 TL.
R: But in the second, he toured a half-round. Why does he have to pay 10 TL**?**
Safiye: Because he always pays 10 TL before starting. He started here but he came in half. But he's paying the fee for a round.

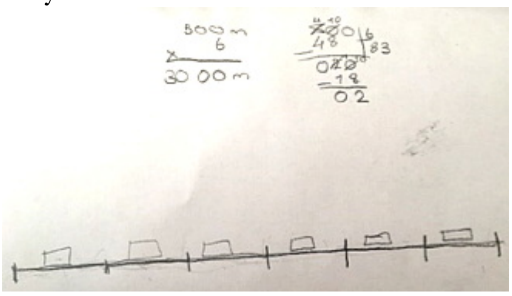
Figure 1: Safiye's solution for Problem 1 and the interview with her

Safiye's problem 2 solving process: Bus stops

Researcher: What did you understand about the problem?
Safiye: I didn't understand it, I'll read it from the beginning.
R: Ok, read it again!
Safiye: Hmm. I get it now.
R: What did you understand?
Safiye: There are 500 metres the road at the bus stops. Someone, who is called Feyza, is going by 6 stops. At the first stop; hmm, I forgot it…!
R: Read it aloud if you want.
Safiye: (She read the problem aloud.)
R: Can you tell me the problem now?
Safiye: They have placed bus stops at 500 m intervals. To get on the bus from a stop….
R: You can look at the question.
Safiye: She got on the bus from the 1st bus stop and off at the 6th stop. In this case, how many kilometres did Feyza go by the bus? Hmm!
R: What did you write there?
Safiye: Well, they placed it as one bus stop on the way. I wrote the length of the road (500 m). She got off at 6th stop. So how many metres did Feyza go by bus? I will multiply.



R: How did you find it?
Safiye: You know; each stop on the road is 500 metres On the way, every 500 metreees, hmmmm! He got on the first stop and off at 6th stop. So I multiplied it by 6. Since each way is 500 metres, she got off at the 6th stop. I didn't need to add and to write "500" six times. Because I have multiplication; it's shorter.
R: Are you sure about your answer? Do you want to check?
Safiye: I can think about it once again.
R: Okay.
Safiye: (After checking the process) I think so. I am sure.
R: Do you want to solve this problem by drawing, as well?
Safiye: Sure.



Safiye: (She drew 6 lines first and then placed the bus stops between the lines.) Let's say this is the first stop. It is the stop 2 up here, the stop 3 up there... stop 6 up here.
R: Do you want to read the problem again?
Safiye: Sure.
Safiye: (After reading the problem again) Aha! they placed them at 500 metres intervals. (Showing the spaces between the rectangular stops she drew). Then it's not 6 but 5. But there are six stops. Okay, then it is true (3000 m).
R: How can we be sure that the result is correct?
Safiye: (After reading the question again.) Since the entire road is 500 metres, we divide 500 by 6.
R: (Pointing the vertical lines) What do the lines you drew in the figure mean?
Safiye: The places in front of stops.
R: What did you find out with this process?
Safiye: The road in front of the stops.
R: What does it want from us?
Safiye: How many metres did she go?
R: So?
Safiye: Hmm. I couldn't make a complete decision.

Figure 2: Safiye's solution for Problem 2 and the interview with her

Safiye's solution for Problem 2 and the interview with her are given in Figure 2. Safiye read Problem 2 silently. She did not use strategies such as highlighting important points during the reading, noting those given, circling, and drawing figures. Safiye failed to understand the problem after reading quietly. The researcher asked her to read the problem aloud, and Safiye was able to explain the problem by looking at it after reading aloud. These findings demonstrate that Safiye had difficulty in understanding the problem. Then, Safiye began to make arithmetic operations multiplying "6" by "500" metres and calculated "3000". She checked only the arithmetic operation to ensure that the conclusion was correct, rather than making any relational control over the data. Safiye performed the arithmetical operation correctly, but she could not reach the correct conclusion, because she considered the number of stops rather than stop intervals. When asked, she stated that each stop was "500" metres and that Feyza got off at the 6th stop, therefore she had to make multiplication as a short way of addition. This finding shows that Safiye did not read the problem carefully, or that she could not determine the appropriate strategy to solve the problem, as she routinely thought about organising what she read, classifying, and seeing the relationships between the given data.

When Safiye was asked to solve the problem by drawing a figure, she first drew 6 lines and placed the bus stops in the middle of the two lines as a rectangle. After positioning the last bus stop, she added 1 more line. She expressed "*Then it is not 6, it becomes 5*" by showing the intervals between the rectangular bus stops in the figure she drew. Then she stated that the number of stops is 6 and that the conclusion of 3000 m is correct. Safiye tried to make sense of the problem through the figure that she drew, but she failed. These findings show that she may be a little distant from routine thinking when illustrating the solution. When asked whether she was sure about the accuracy of the solution, she read the question again and divided "500" metres by "6". It was once again observed that Safiye performed the arithmetic operation correctly. She stated that this operation helped her find the way where the bus stops are located. These findings also indicate that Safiye could not determine the appropriate strategy to solve the problem, as she routinely thought about organising that she read, classifying and seeing the relationships between the given data.

Burak's problem 1 solving process: Bike trail

Burak's solution for Problem 1 and the interview with him are given in Figure 3. Burak read Problem 1 silently. He did not use strategies such as highlighting important points during the reading, noting those given, circling, and drawing figures. However, he was able to express the problem correctly in his own sentences. Burak's statements revealed that he comprehended the given data successfully, but not the problem. Burak was able to make reasoning quickly and then reached the conclusion as "15" by making operations from his mind without using paper and pencil. Burak was asked to show on paper the operations he used, which showed that he performed the arithmetical operations correctly. Burak checked operational errors to ensure the accuracy of the conclusion, but he did not examine data for relational control. Burak failed to reach the conclusion by making a simple proportion.

This finding shows that Burak did not carefully read the problem, or that he could not determine the appropriate strategy for solving the problem, as he thought routinely about organising, classifying, and seeing the relationships between the given data. Burak did not want to draw a figure about the problem, because he was sure that his solution was correct. Since Burak could not interpret the statement "*A full tour fee is paid at the beginning of* *each tour"* in the problem, he did not initially realise that a full tour fee should be paid for a half tour. However, when the Researcher asked him questions about the concept of half-round by stressing the statement "*A full tour fee is paid at the beginning of each tour"*, Burak was able to reason about the problem and reach the correct conclusion.

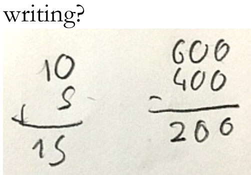
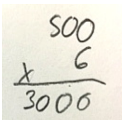
Researcher: What did you understand from the problem? **Burak**: He pays 10 TL in a round. That is 400 metres It says; How much TL does he pay at 600 m? **R:** What can we do here? **Burak**: First we have to do one. He paid 10 TL and then has 200 m left. If he pays 5 TL for 200 m, that's the half, he pays 15 TL. **R:** How can we be sure that the answer is correct? **Burak**: I'm sure. I did the operation correctly. **R:** Can you show me how you solved it by writing?  **Burak**: Yes. Nooow! "We paid 10 TL", he says before starting. "He pays a full tour fee", so what does it mean? He's paying just 10 TL. How much TL should he pay to ride a bike for 600 m? We've made our operation. We subtracted 400 from 600 = 200. We paid for a round. But before starting, he pays 10 TL. And he rides 200 m, that's 5. If we add it, it is 15.	**R:** Do you want to solve the problem by drawing? **Burak**: No, it's not necessary. I think that's right. **R:** How many rounds did he tour by bike? **Burak**: One round. **R:** How much TL does one round cost? **Burak**: 10 TL. **R:** Why did he pay 15 TL? **Burak**: But he says that he pays 10 TL before starting. **R:** What does "A full tour fee is paid at the beginning of each round" mean? **Burak**: 10 TL, I think. **R:** How many rounds does 200 m correspond? **Burak**: A half-round. **Researcher:** Does one have to pay a fee for half a round? **Burak**: Yes. If we pay for a round, we have to pay half a round, too. **R:** It says "A full tour fee is paid at the beginning of each round". Is there anything on half a tour here? **Burak**: Ohh! Then we'll pay 10 TL again. We won't pay 5 TL for 200 metres We will pay 10 TL before starting the second round. We will add 10 and 10 = 20 TL.

Figure 3: Burak's solution for Problem 1 and the interview with him

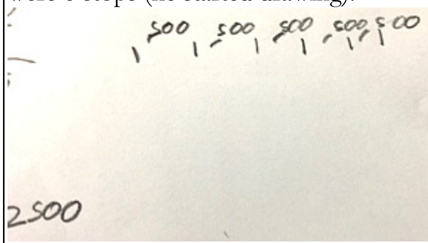
Burak's problem 2 solving process: Bus stops

Burak's solution for Problem 2 and the interview with him are given in Figure 4. Burak read Problem 2 silently. He did not use strategies such as highlighting important points during the reading, noting those given, circling, and drawing figures. However, he was able to express the problem in his own sentences. Burak's verbal statements might be an indicator that he understood the problem very well. Burak tried to solve it by making operations from his mind and calculated the result as 3500. While calculating, Burak said "500", and counted his first finger as "1000" to reach this conclusion. When asked to solve the problem by writing, he multiplied "6" by "500" metres and reached the conclusion of "3000". Burak compared what he wrote on the paper with the operation in his mind and stated that it was correct. Burak only checked for operational errors to ensure the accuracy of the conclusion; he did not make any relational control over the data. Burak performed the arithmetic operation required for the solution of the problem correctly, but he did not reach the correct conclusion because he considered the number of bus stops rather than stop intervals. Burak explained why he did this operation by drawing a figure and noticed that the station intervals were 500 metres Therefore, he calculated the correct result. According to these findings, Burak was initially unable to interpret the statement "*…the bus stops were placed at 500 metres intervals"* and got an incorrect conclusion. However, while trying to explain the result, he started to draw a figure, realising his error with the help of the figure he drew, and reached the correct conclusion. This finding shows that Burak's reorientation about the result by the Researcher and figure drawing can be effective in solving problems correctly.

Burak: 500, 1000, 1500… 3500 metres (calculating with fingers.)
R: How did you find 3500?
Burak: I counted.
R: Can you tell me what you understand from the problem?
Burak: She got on the bus at the first stop and off at the sixth stop. If I count by adding 500; 500, 1000, 1500 ... 3500.
R: Can you solve the problem in writing?
Burak: Yes.



Burak: Wait a minute, it turned out wrong.
R: Which was wrong?
Burak: My counting was wrong. 3000 is true.
R: Are you sure the answer is correct?
Burak: Hmm. It's zero. 6 times 5 is 30. I'm sure.
R: Why did you do this?
Burak: Because it is said that between the first stop and the second stop…. One minute… He said 500 m between two stops but… if there were 6 stops (he started drawing).



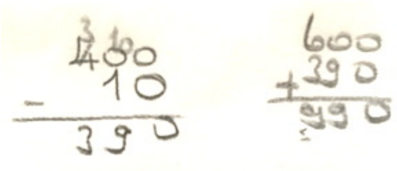
Burak: If we add; 500, 1000, 1500, 2000, 2500.
R: Which one is true now? We found 3500, 3000, 2500.
Burak: 2500.
R: Why?
Burak: Because he said, "between the two stops".
R: But there is no expression of the problem as "between the two stops".
Burak: So, when you go 500 m, there is one stop in that place. When you go to 500 m, there is one stop in that place. That is, between the stops.

Figure 4: Burak's solution for Problem 2 and the interview with him

Özlem's problem 1 solving process: Bike trail

Özlem's solution for Problem 1 and the interview with her are given in Figure 5. Özlem read Problem 2 quietly. She did not use strategies such as highlighting important points during the reading, noting those given, circling, and drawing figures. It can be concluded from Özlem's expressions that she had difficulty in understanding the problem. Özlem thought that she had to subtract 10 from 400 and she did this operation arithmetically correctly using paper and pencil. She answered the question as to why she did this operation, *"Because he rode bike 400 metres because he paid 10 TL".* The Researcher asked what "*He pays a full tour fee at the beginning of each round*" meant, and she answered "*He pays 10 TL each time when he rides a bicycle. …before he starts riding*". Afterwards, she stated that she subtracted 10 from 400 and added 600 to the calculated result. Therefore, Özlem failed to solve the problem even though she performed the arithmetic operations correctly. Nonetheless, she could not explain why she did these operations. In addition, Özlem only checked operational errors to ensure the accuracy of the conclusion and did not perform any relational control of the data.

Researcher: What did you understand from the problem?
Özlem: We need to subtract 10 from 400.
R: Why?
Özlem: Because he rode a bike for 400 m… because he paid 10 TL. Then, he paid a full round fee at the beginning of each round.
R: What does the expression "He pays a full round fee at the beginning of each round" mean?
Özlem: So he pays 10 TL whenever he rides a bike. I mean, before starting to ride.
R: What are we going to do then?
Özlem: I subtracted 10 from 400 = 390. Then I add 600 to 390.



R: Why?
Özlem: (No answer).
Researcher: What does a round mean?
Özlem: (Making a circle with her hand) Going once.
R: How many rounds does he tour here?
Özlem: (No answer)
R: What did you find now?
Özlem: 990.
R: How can we be sure it's correct?
Özlem: (After checking her operations she made) Correct.
R: Can you solve this problem drawing?
Özlem: Okay.
R: Can you tell me your solution through the shape?
Özlem: The bike is going.
R: How far is it going?

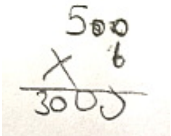
Özlem: 400 metres.
R: What does he do next?
Özlem: He pays 10 TL
R: Does he pay after or before he goes?
Özlem: He pays before.
R: Where does the bike come back after leaving?
Özlem: To the same place.
R: How far does it go when it comes back to the same place?
Özlem: 600.
R: TL
Özlem: 400.
R: TL
Özlem: 1 round.
R: TL
Özlem: 400.
R: TL
Özlem: He pays his money at the beginning of each round.
R: TL
Özlem: 10 TL.
R: TL
Özlem: 990 metres.
R: TL
Özlem: (After thinking for a while) 990 TL.
R: How can we be sure that the answer is correct?
Özlem: (Referring to the operations she made) I have just checked it! I'm sure.

Figure 5: Özlem's solution for Problem 1 and the interview with her

When asked to solve the problem by drawing a figure, she could not correctly interpret the data given on the figure although she correctly positioned the problem data on the figure. Despite the questions of the Researcher to solve the problem, she could not provide the correct response.

These findings may indicate that Özlem did not read the problem carefully or that she did not succeed in organising, classifying, and seeing the relationships between those reads; that is, she could not understand the problem thoroughly and determine the appropriate strategy for solving it. The findings might also indicate that the context of the question was unfamiliar to her, that she may have read it carefully but not understood it, or that she simply struggles with mathematical concepts that go beyond calculating according to a given formula. She appears to understand that numbers can be added and subtracted, but not understand the reasons why you would choose to use these operations.

Özlem's problem 2 solving process: Bus stops

<table>
<tr>
<td>Researcher: Can you tell me what you understand from the question?
Özlem: Feyza was located at a distance of 500 m to go to Bayburt…. (Feeling confused) So one stop at 500 metres interval.
R: What can we do here?
Özlem: I will multiply 500 by 6.
R: Why?
Özlem: Because if I multiplied 500 by 1, it would be 500 again. So, I multiplied it by 6.


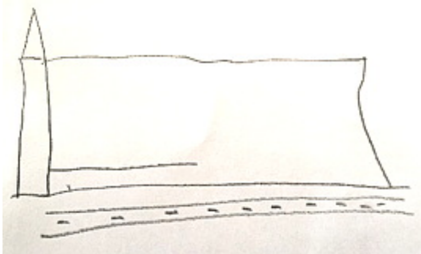
R: What would happen if it were 500 again?
Özlem: (No answer)
R: You multiplied it by 6, so what?
Özlem: Let's find out how far Feyza has gone.
R: Are you sure the result is correct?
Özlem: I'm sure. 6 times zero is zero. 6 times zero is zero; 6 times 5 is 30.</td>
<td>R: Can you solve this problem by drawing?
Özlem: Okay.

R: Which are the stops in this picture?
Özlem: (Pointing to the tower-like shape) That's it!
R: Is there only one stop?
Özlem: Yes.
R: Does the problem mention only one stop?
Özlem: Yes.
R: What do we do now?
Özlem: (No answer).</td>
</tr>
</table>

Figure 6. Özlem's solution for Problem 2 and the interview with her

Özlem's solution for Problem 2 and the interview with her are given in Figure 6. Özlem quietly read the problem. She did not use such strategies as highlighting important points during the reading, noting those given, circling, and drawing figures. It seems from Özlem's verbal expressions that she had difficulty in understanding the problem. Özlem multiplied 6 by 500 metres and reached the result of "3000". So, Özlem could not solve the problem correctly. She explained, "*Because if I multiplied 500 by 1, it would still be 500. So I multiplied it with 6"*. In order to ensure the accuracy of the conclusion, Özlem only checked the operational errors and did not conduct any relational control over the data. This finding shows that Özlem could not relate the number 6 used in the operation to the number of bus stations and make sense of what was provided in the problem; therefore, she failed to understand the problem. When asked, she could not draw a figure expressing the problem and position all pieces of given information on the figure. This finding might be attributed to the fact that her failure in understanding the problem made it difficult for her to visualise the problem.

Discussion

Non-routine problems are the problems that due to their nature need to be carefully read and interpreted from various aspects. This study has indicated that the three participant students did not use the strategies that could be used while reading the problems, such as highlighting the important points, noting down the given, circling, and pausing at certain intervals to think. So, it was revealed that only the student with a moderate level of mathematics achievement was able to express both problems with his own sentences and successfully comprehend the problems. The student with a high level of mathematics achievement failed to express the second problem, whilst the student with a low level of mathematics achievement could not express either problem in her own sentences. Pólya (2004) stated that understanding the problem was the first step for its solution. In this step, students may note down the given, draw a figure, highlight the important points or separate out various parts of the problem (Alvi & Nausheen, 2019). The fact that the students do not use reading strategies can be evaluated as a deficiency in their effort to understand the problem.

Strategies used in reading problems enable the reader to communicate with the text, relate to life, and establish meaning (Akkaya, 2011). The literature on reading suggests that reading strategies reduce reading errors and improve reading and comprehension of reading (Lee, 2017; Lipp & Helfrich, 2016; Kaman, 2012). Hence, the students with difficulty in comprehending the problems could be counseled to use reading strategies to overcome this problem. Hite (2009) concluded that the use of reading strategies helps students develop their problem-solving skills. In addition, Büyükalan, Filiz, Erol and Erol (2018) found a positive relationship between the frequency of using metacognitive reading strategies, and success in solving non-routine problems. Likewise, Ulu, Tertemiz and Peker (2016a) concluded that training on reading comprehension skills increases students' ability to solve non-routine problems.

The current research has also shown that the students at all three levels performed correct arithmetic operations related to the numbers given in the problems. This result was expected only from the students with high and moderate level of mathematics achievement. So, it was surprising to see that the student with a low level of mathematics performed those operations successfully. This situation might be an indicator that competence in arithmetic operations is not sufficient for mathematics achievement and that higher cognitive skills, such as reading comprehension, relating the given, and multi-faceted thinking, of the students with a low level of mathematics achievement should be improved. Özyıldırım, Gümüş and Umay (2017) noted that operations are the means for the implementation of mathematical thinking and coming to conclusions and that they are the *sine qua non* of mathematics. Competence in operations is necessary but is not sufficient. These findings indicate that the participant students used the strategy named "*solve first, think later*" by Hegarty, Mayer and Monk (1995). In this strategy, understanding the problem lags behind the arithmetic operation.

The findings reveal that the three students checked the arithmetic operations they performed, to ensure the accuracy of the results, while they ignored the essentials of the problem such as the use of the given data required for the solution, accuracy of data relationship, and fiction of the solution. This situation might have led the students to be convinced about their incorrect responses and decreased their problem-solving success. Yoshida, Verschaffel and De Corte (1997) attributed this to teaching practices whereby students' attention is given fully to operational competencies rather than interpretation skills. Özsoy (2005), on the other hand, reported a positive significant relationship between fifth-grade students' ability to control the solution of the problem and their mathematics achievement.

The study has also revealed that the arithmetic operations performed by the participant students were not initially necessary for correct solution of the problem, although they were correct. In other words, the students of all achievement levels in mathematics failed to solve the non-routine problems correctly in their first attempt, primarily because they interpreted them as routine problems. Hence, this finding largely coincides with Chakko (2004) who reported that students thought of non-routine problems as routine problems. Likewise, it conforms partially with Artut and Tarım (2006) and Aydın, Memnun, Dinç, Arsuk and Meriç (2019), who reported that 7th graders largely failed to solve non-routine problems. The current finding is also in line with Krawec (2014); Boonen, van der Schoot, van Wesel, De Vries and Jolles (2013); Palm (2008); and Xin, Lin, Zhang and Yan (2007), who found that students had difficulty in solving these problems. This may be attributed to the fact that they failed to find the appropriate strategy for the solution of these problems, as they failed to understand the problems and had not encountered them frequently. This study concludes that students with high and low-level mathematics achievement have difficulty in understanding the problems they read. This particular finding conforms to the existing literature reporting a positive correlation between reading comprehension and non-routine problem-solving skills (Başol, Özel & Özel, 2018; Ulu, Tertemiz & Peker, 2016a; Matel, 2013; Hite, 2009). Ulu, Tertemiz and Peker (2016b) concluded that fifth-grade students made more mistakes in reading comprehension while solving non-routine problems.

The study indicated that the students who failed to solve the problems in their first attempt could begin to position the given data on the figure when they were asked to solve it by drawing a figure. Safiye with a high level of mathematics achievement was able to solve the first problem correctly after thinking about the figure. She was also able to justify the solution even though she could not reach the correct result after thinking about the figure in the second problem. Burak, with a moderate level of mathematics achievement, on the other hand, was not eager to draw a figure for the first problem but could reach the correct result with explanatory questions from Researcher. Burak was also able to reach the correct conclusion after thinking about the figure in the second problem. Özlem, with a low level of mathematics achievement did not manage to solve both problems by drawing figures. According to these findings, it could be claimed that the use of drawing strategy in the solution of non-routine mathematical problems may contribute to obtaining the correct solution to the problem. Ulu (2008) and Olkun, Şahin, Akkurt, Dikkartın and Gülbağcı (2009) concluded that students tend to solve non-routine problems without drawing a figure. Elia, Gagatsis and Demetriou (2007) and Ulu and Akar (2016) concluded that the use of visuals increased students' success in solving non-routine problems. McLure (2020) concluded that students' multiple-representations supported conceptual understanding. However, Diezman (2005) and Elia et al. (2007) stated that visualising problems may result in errors such as incomplete or incorrect information.

Conclusion

This study has indicated that the three participant students did not use the strategies that could be used while reading the problems; failed to solve the non-routine problems in their first attempt even though they performed the correct arithmetic operations; that to ensure the accuracy of the results they checked only the arithmetic operations they performed, and that the use of a drawing strategy in the solution of non-routine mathematical problems contributed to reaching the correct solution of the problem.

Accordingly, it may be suggested that the students represented in this research should be provided with training in reading strategies, and that the development of problem comprehension skills and competence in relating the given data should be prioritised over the teaching of arithmetic processing skills in mathematics courses. It may also be suggested that activities designed to improve the students' reading comprehension and non-routine problem solving skills should be better integrated into classroom teaching. Finally, it might be concluded that encouraging students to diagrammatise when solving math problems could increase their math achievement.

As this study was limited to fourth-grade students' non-routine problem solving skills, future studies could investigate students in other primary school grades. Being designed as a case study, this research did not aim to reach generalisable results. More generalisable results could be obtained in further studies using other research designs, such as the scanning model and quasi-experimental research. Future studies might also explore primary school students' non-routine problem-solving skills with regard to demographic variables such as gender, age and pre-school training.

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