Teachers’ and learners’ experiences for guiding effective teaching and learning of mathematics word problems

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Understanding the challenges pertaining to the teaching and learning of mathematics word problems is important in order to formulate effective strategies that will address the challenges. The qualitative case study reported in this article describes the teachers’ and the learners’ experiences regarding mathematics word problems. Data were collected through focus group discussions and reflection sessions, through the use of the free attitude interview technique used to initiate the conversations. Thematic analysis was used to analyse data. Analysis of data revealed challenges related to lack of English proficiency, limited knowledge of mathematical vocabulary, the effects of “out of context” meanings and lack of understanding mathematical language and structure to be the sources of difficulty for teaching and learning mathematics word problems. Findings of the study suggest the need for challenges to be understood in context in order for meaningful possible solutions to be formulated. Thus the learners’ experiences should be regarded as important guidelines for informing better teaching of mathematics word problems.

Introduction

Mathematics word problems (MWPs) form part of realistic mathematics education which contributes to problem solving in the sense that they provide rich, “realistic” situations that play a significant role in the learning process (Van den Heuvel-Panhuizen & Drijvers, 2014). Solving mathematics word problems has long been an integral part of mathematics education (Csíkos & Szitányi, 2020). It also helps learners to develop the skill of knowing when and how to apply classroom mathematical knowledge in their daily life problems. These problems are regarded as important component of mathematics education, which amongst others; equip learners with problem solving skills that are necessary in other subjects (e.g. physical science, accounting, etc.) (Boonen, de Koninck, Jolles & van der Schoot, 2016). MWPs thus play a vital role in other fields and for this reason, they need to be taught well in order to lay a good foundation. Although they play such an important role in enabling learners to connect and apply their mathematical knowledge to real life problems, they prove to be difficult for most learners (Morton & Qu, 2013; Pearce, Bruun, Skinner & Lopez-Mohler, 2013).

Some learners are unable to interpret MWPs, especially when they cannot visualise the concepts (Moleko, 2019). Some learners experience difficulty in terms of understanding the questions and this inhibits them from converting the word problems into mathematical equations (Sepeng & Sigola, 2013). Learners rely on keywords to solve the MWPs and this often leads to incorrect translations (Salemeh & Etchells, 2016). While word problem solving requires the learners’ ability to read, process, and solve mathematical situations (Goldberg, 2003), this ability is usually not cultivated by teachers in learners, due to their tendency to encourage “mechanical” ways of solving mathematical problems (Sepeng & Webb, 2012).
Research indicates the significance of understanding mathematical vocabulary and providing mathematics teaching that equips learners with explicit knowledge of mathematical vocabulary and register (Salemeh et al., 2016). Although this is important, Moleko (2019) found that many teachers do not provide explicit teaching of mathematical vocabulary and this creates confusion and impedes learners’ understanding of the content that is embedded within the MWP. Kotsopoulos (2007) also found that most learners experience challenges in solving MWP s, especially when words borrowed from everyday language are used, where they have to attach different meanings depending on the context in which these words are used. Learners may also show an inability to visualise the MWP s, which hampers problem solving (Teahen, 2015). As a result, lack of visualisation skills leads to misinterpretation of questions and attainment of incorrect solutions (Yeo, 2009).

Previous studies have been conducted on the teaching and learning of MWP s with specific focuses on the use of the different strategies (Sajadi, Amiripour & Rostamy-Malkhalifeh, 2013; Morin, Watson & Heste, 2017); sources of difficulty and the effects of language on word problem solving (Gooding, 2009; Sepeng & Madzorera, 2014; Salemeh & Etchells, 2016), and exploring learning of word problems (Bruun, Diaz & Dykes, 2015; Mahofa, Adendorff & Kwenda, 2018). In all of these studies importance was placed on the teachers as reliant sources of information to explain what the problems are and to recommend solutions to the identified challenges. Methodologically, there is paucity of studies that take into account the learners’ experiences thereby bringing two voices, teachers and learners, together in an endeavour to create a platform for both to participate equally in the research project (Sepeng, 2014). Therefore, this article seeks to bring the experiences of both teachers and the learners together, to help guide effective teaching of MWP s.

In this article, we argue that unless mathematics teachers fully understand the challenges or sources of difficulty related to MWP s, they will not be able to devise the strategies that will respond to the needs of the learners. We also argue that unless the learners’ experiences are not fully considered in the planning of teaching MWP s, that solutions directly addressing their challenges will not be formulated.

The study thus has at least two theoretical implications: firstly, the findings contribute to the expansion of literature (teaching and learning theories) to promote issues of inclusivity and sustainability in teaching and learning. Secondly, the study will serve to transform how planning for teaching and learning of MWP s should be carried out. The study thus proposes a “new” idea of a learner-driven teaching planning approach which should be viewed as one important initiative to drive the agenda or culture of inclusive planning.

The message that is conveyed in this article is that the challenges pertaining to the teaching of MWP s should be understood not only from the teachers’ perspectives, which are informed by their own experiences. Learners’ experiences also should serve as basis for understandings to be established. Much of the work in this study relies on teachers’ and learners’ reflections on classroom mathematics discourses and mathematical modelling. The emphasis in this article is that the teaching of MWP s should be intentional and thus must consider the underlying sources of difficulty in order for meaningful
learning to take place. The implications of the study are situated in debates on promoting the improvement of teaching and learning mathematics, as highlighted by results of some international benchmark assessments, such as *Trends in International Mathematics and Science Study* (TIMSS). In line with all this therefore we ask the question:

How can the teachers’ and learners’ experiences inform the effective teaching and learning of mathematics word problems?

**Literature review**

Mathematics word problems are defined differently by various authors. Some authors refer to them as problems that portray real life situations (Verschaffel, Van Dooren, Greer & Mukhopadhyay, 2010). Chapman (2006) referred to mathematics word problems as mathematical problems which are presented in the form of a ‘real world’ or social/cultural context (real or imagined). Verschaffel, Greer and De Corte (2000) defined them as textual portrayals of situations presumed to be comprehensible to the reader, within which mathematical problems can be contextualised. For the purpose of the study reported in this article, the definition provided by Kasule and Mapolelo (2013) is used. According to these authors, MWPs are mathematical problems whose content is epitomised in the form of a word-based story. They are real-life problems that require mathematical knowledge for solutions, and usually are not presented as equations that are easy and ready to be solved, but rather as complex textual representations that must be interpreted symbolically, manipulated, and solved. They are an important part of the mathematics curriculum because of their ability to promote realistic mathematical modelling and problem-solving (Van den Heuvel-Panhuizen et al., 2014). MWPs are an important part of mathematics curricula and should be allocated time in mathematics classes (Ergen, 2020).

MWPs enable learners to make connections between classroom mathematical knowledge and knowledge from everyday life (Sepeng, 2013). According to Bernardo (999), MWPs are the most challenging problems to solve in mathematics education. This is because they do not only necessitate the use of numbers to be executed, but also require the skill to identify, understand and interpret the given problems (UNESCO, 2003). The complexity of these problems is often aggravated by its requirement to use English text to discover missing information (i.e. the unknown) (Fuchs et al., 2008), which is what most learners fail to do since they lack English proficiency (Essien, 2013). Adding to these challenges is the issue of teachers, who just like the learners, also grapple with the mastery of MWPs (Seifi, Haghverdi & Azizmohamadi, 2012).

More than two decades ago, Braselton and Decker (1994) also attributed the complexity of teaching MWPs to learners’ lack of reading skills. According to Gooding (2009), learners who have difficulty reading English as the language of learning and teaching (LoLT) usually have trouble solving MWPs. Fuchs and Fuchs (2002) added that learners who struggle with reading skills find word problems challenging to execute, usually performing worse than learners experiencing only mathematics difficulties, or learners experiencing neither reading nor mathematics difficulties. On the other hand, Salihu, Aro
and Räsänen (2018) noted that reading difficulties exacerbate rather than cause mathematics difficulties.

MWP\text{s} require one to have a deeper understanding of how the mathematical key terms are applied, since they enable one to understand what the problem is about. In their study, Vula and Kurshumlia (2015) revealed that many learners struggle to understand and solve the MWP\text{s}, because they have a limited knowledge of mathematical vocabulary and register. Sepeng and Sigola (2013) concurred and added that lack of understanding of mathematical vocabulary and register causes misapplication of the appropriate mathematical operations. Apart from this, visualisation of MWP\text{s}, even though deemed to be an essential skill for successfully solving the problems, is revealed in the literature to be lacking for most learners. As a result, they misinterpret the questions and thus obtain incorrect solutions (Yeo, 2009). Teahen (2015) added that the inability to visualise MWP\text{s}, and the mathematics content involved, exacerbates the learner struggle to solve these problems. On the other hand, the use of polysemous words were also found to be the cause of translation ambiguities, which mired word problem solving. According to Reynders (2014), polysemous words cause misinterpretations, predominantly in instances wherein learners have limited knowledge of the application of the different words with similar meanings.

Although the use of the learners’ home languages benefit learners, especially those that lack English proficiency, most teachers still prefer to teach MWP\text{s} only in English which in many instances, creates challenges for many learners especially those who are not proficient with the English language. According to Setati (2008) the choice of teaching primarily in English is mainly influenced by the high level of esteem that is placed on the English language, which many people regard to be a “great success or achievement indicator” of the person’s level of education. Although teaching learners in English may be beneficial, the fact that most learners do not have English proficiency and that they do not fully understand it as a medium of instruction (Barwell, 2003) requires them to be provided with academic support.

**Home languages and implications for learning and teaching**

There are eleven official languages in South Africa and the constitution gives provision for learners to learn in any of the eleven official languages of their choice. Amongst these 11 official languages, Sesotho, IsiZulu and IsiXhosa, which are highlighted in this study, are included and they are referred to as “home languages”. They are referred to as the main languages, because they are languages in which the learners and the teachers are most fluent in, and they often speak them more than any other language (Essien, 2013). English is referred to as the language of learning and teaching (LoLT) and it is mainly used in teaching and learning spaces in South Africa and may or may not be the dominant language of learners or teachers. Research shows that learners who come from homes where the LoLT is the only language spoken at home are accustomed to the linguistic structures they come across in the mathematics classroom, and learn mathematics better than those for whom LoLT is not their home language (Barwell, 2003; Cuevas, 1984).
On the other hand, research also shows that learners whose home language is not the LoLT struggle to learn mathematics since they have to deal with the additional restraint of not being fluent in the LoLT (Adler, 2006; Barwell, Barton & Setati, 2007; Halai, 2004; Setati, 2002). In a classroom comprising learners whose home language is not the LoLT and who are not proficient in the LoLT, teachers are faced with a triple challenge of striking a balance between attention to mathematics, English (LoLT) and mathematical language (Barwell, 2009). Learners whose first language is not the LoLT need to be supported through the use of home languages to enable them to learn (Halai, 2004).

**Theoretical framework**

Critical emancipatory research (CER) is a transformative paradigm espoused here as a lens which advocates the inclusivity in the research process. CER encourages the creation of opportunities for people who experience the challenges to freely express their different perspectives. CER thematises issues of transformation, empowerment and endorsing collaborative working in addressing the issues in meaningful contexts (Nkoane, 2009). Epistemologically, researchers who apply this framework in their work believe that knowledge is true if it can be turned into practice that transforms the situation and empowers the lives of the people (Al Riyami, 2015). De Klerk and Palmer (2020) maintained that when in-service teachers surpass their professional knowledge base, it empowers them to transform learning spaces.

This study included both teachers and learners. Although the teachers are more experienced than the learners, they were both involved in the study because of their experiences. The teachers were expected to share their experiences regarding the teaching of word problems whereas the learners were expected to share their experiences in terms of learning to solve word problems. Through collaborative discussions, the participants identified the challenges and recommended solutions.

**Method**

This qualitative case study was conducted in one of the schools in Thabo-Mofutsanyane district. A purposive sampling was used, involving five mathematics teachers (including the Head of Department for Mathematics) who had 5 to 18 years teaching experience; 9 learners (aged 16 to 18 years) from grades 10 to 12 (3 learners from each grade). The school represented a bounded case from which in-depth data were generated through focus group discussions and reflection sessions (Creswell, 2014). The meetings (focus group discussions and reflection sessions) were conducted in the months of September and October 2017 and were audio recorded. The school uses English as a medium of instruction, though it was dominated by Sotho, Xhosa and Zulu speaking teachers and learners. Therefore there were three home-languages that were used in the school. Two focus group discussions and three reflection sessions with each meeting lasting for 2 hours were conducted. One focus group discussion was with the five teachers and the other with the nine learners. The two reflection sessions were conducted for the teachers to reflect upon their own practices and one conducted for the learners to reflect upon their learning experiences. The research data collection was conducted in English.
Ethical clearance was obtained from the University of the Free State. Permission to conduct the study was also obtained from the Department of Education and the school principal wherein the study was conducted. Participants signed consent forms that emphasised their confidentiality and rights. Since the students who participated were minors, their parents were requested to sign the assent forms for their children to take part in the study.

Data collection procedure

The *Free Attitude Interview* technique (FAI) was used to initiate the conversations and discussions (Buskens, 2011). FAI is a data generation technique that allows people (participants) to talk as they would in a normal conversation (Buskens, 2011). Its “normal discussion/ conversation” nature promotes unrestricted and open engagement of the participants (Hlalele & Tsotetsi, 2014). Only one comprehensive question is usually asked when this technique is used to initiate conversations. The meetings and reflection sessions that we had, comprised discussions around the challenges pertaining to the teaching and learning of word problems in a mathematics classroom. Solutions to the challenges were also discussed during these meetings. The discussions were carried out in the form of conversations, reflections and demonstrations/ illustrations. To avoid misinterpretation of the spoken words, member checking was carried out.

Data analysis

Data were analysed through a thematic analysis technique. Thematic analysis is a technique that is used to identify, analyse, and interpret patterns of meaning ('themes') within qualitative data (Clarke & Braun, 2015). It provides accessible and organised procedures for generating codes and themes from qualitative data (Vaismoradi, Turunen & Bondas, 2013). The technique made it possible for us to identify the emerging themes and to organise them according to challenges and solutions. In line with this, the main and sub-themes were highlighted.

Ensuring rigour

Credibility is all about ensuring that the study measures what it is intended and that it is a true reflection of the social reality of the participants (Hadfield, Hutchings & de Eyto, 2018). In order to address the issue of credibility in this study, the “prolonged engagements” were conducted through the FAI technique. The FAI technique afforded the participants opportunities to say more than they would in closed-ended questionnaires and this resulted in more clarity being provided and issues being discussed in depth. Furthermore, the data were generated from multiple instruments and that enabled the researcher to perform triangulation which in turn helped in facilitating validation of data through cross verification from multiple sources.

Transferability refers to the ability of the findings to be transferred to other contexts or settings (Barusch, Gringeri & George, 2011). To address the issue of transferability in this study, the findings were interpreted in an explicit manner, with an intent to enable the
readers to fully understand the phenomenon being explored and also make it easy for them to assess whether those findings are transferable to the other contexts.

Dependability ensures that the process is described in detail to enable another researcher to reproduce the work (Hadfield et al., 2018). The study was conducted in a systematic manner, with all the details provided on processes followed to make it easy for the work to be reproduced by another researcher.

Confirmability is akin to objectivity in quantitative studies (Cypress, 2017). The essential goal of confirmability in qualitative studies is to minimise researcher bias by conceding researcher’s predispositions. To minimise bias and falsification of data, member checking was conducted in order to ensure that data were analysed and the findings interpreted in a manner that clearly and correctly portrayed the participants’ ideas and opinions.

Findings and discussion

The following sections provide analysis of the data drawn from the reflection sessions and focus group discussions, as well as discussion of the findings. Some extracts are used to indicate what the participants regarded as the challenges pertaining to the teaching and learning of word problems. Solutions to the challenges are also highlighted in the subsequent sections.

Reflection sessions

These were established for the participants to witness and reflect upon their own practices. These sessions gave the participants opportunities to think about their own experiences and to explore them in greater depth. The sessions gave the teachers opportunities to realise the challenges they were all faced with, in terms of teaching word problems. The teachers were also empowered through these sessions by learning from each other’s described way(s) of tackling the challenges pertaining to the teaching of mathematics word problems.

Focus group discussions

Focus group discussions were conducted to gain an in-depth understanding of the issues. During the focus group discussions, teachers were given opportunities to discuss the issue (phenomenon) in-depth without necessarily having to specifically make reference to their experiences but to engage deeply on the issues and to recommend possible solutions.

Lack of English proficiency

During a focus group discussion when English language as a medium of instruction was highlighted as an impediment towards the teaching of word problems, the teachers and learners commented as follows:
Teacher 1: Most learners lack English proficiency. They are unable to convert a word problem into variables. For example, eehh… when you have given them the problem, y is equal to three less than the number b, they express it as $y = 3 - b$, instead of writing $y = b - 3$

Learner 3: It is difficult to solve the word problems because we usually struggle to understand English…

Learner 1: In order to master the word problems we need to know how to read and I think our teachers need to teach us how to read these problems.

Teacher 3: I strongly support the idea of learners being taught how to read and this needs to be done on regular basis.

Learner 2: I think it will be much better if our languages are also used.

Learner 4: When we are taught in both our languages and English we are able to understand the concepts…

Teacher 5: We should only use the home-languages to help learners understand and we should refrain from only using them because tests and assignments are usually set in English…

As illustrated in these extracts, lack of English proficiency makes it difficult for learners to comprehend and execute the word problems. As a result, most learners are unable to translate the word problems into easily solvable equations. From the example provided by Teacher 1, namely, “$y$ is equal to three less than the number $b$”, it seems as if most learners were unable to express it algebraically since they expressed it as $y = 3 - b$ which is not mathematically correct. This example shows that learners usually use a “one-to-one, left-to-right linear translation” when they read the MWPs and this form of reading does not always help provide the correct translation, because the order of mathematical concepts when they are expressed in words differs from when the concepts are expressed algebraically. Teacher 1’s statement indicates that lack of English proficiency impedes not only understanding of the word problems, it also hinders procedural fluency. Therefore teachers who teach word problems to learners, without helping them overcome language barriers, make it difficult for learners to solve mathematics word problems.

Some of the solutions were recommended by the participants to address the lack of English proficiency. Teaching learners the mathematical vocabulary as well as engaging them in reading exercises were considered helpful for improving English proficiency. A recommendation came from Learner 1, supported by Teacher 3, that learners should be engaged frequently in reading exercises that involve word problems. The idea of reading and teaching vocabulary is also supported by Rindyana and Chandra (2012) and Seifi, Haghverdi and Azizmohamadi (2012), who emphasised that the understanding of the meaning of vocabulary as it exists in MWPs is an important point for teachers to pay attention to. In support of the idea of reinforcing reading skills, Salihu et al. (2018) further recommended the type of instruction that adequately matches the learning and instructional needs, and has the capacity to improve learners’ achievement and performance, notwithstanding their background characteristics.
In addition to teaching vocabulary, it was suggested that teachers should teach learners the mathematical language structure in conjunction with the English structure. Learners 2 and 4 additionally recommended the use of home languages in order to assist learners who lack English proficiency, which is an idea that Chitera (2009) also espoused. Teacher 5 emphasised though, that the home languages should only be used as “resources or supplements” to support English comprehension and not as “absolute teaching and learning languages”. Teachers should therefore not entirely rely on the use of them since learners are still expected to develop English mastery in order to fulfil the mathematical related tasks that are “always” set in English (e.g. tests and examinations). Barwell (2009) supported this notion and further stated that the significance of using the home languages lies in the fact that most learners in South Africa classrooms are still learning the language of teaching and learning (English) and are still grappling with it (Essien, 2013).

**Limited knowledge of mathematical vocabulary**

During the data collection session / reflection session where limited knowledge of mathematical vocabulary was also highlighted as a challenge towards teaching and learning of word problems, teachers and a learner commented:

Teacher 3: Learners often do not know the mathematical vocabulary. For instance, most of them do not know the meaning of the term “difference”. The term means dissimilar. I remember there was a time when I gave them a word problem where they had to determine the difference between nine and three. Yhooo! some indicated that nine is bigger than three and some indicated that nine is three times bigger than the number three.

Learner 6: I think what makes it challenging for us is that there are many words that we do not know that confuse us.

Teacher 2: I agree that we need to teach learners vocabulary. This will help them solve the mathematical problems.

Teacher 4: Maths vocab is important! But we also have to make sure that we pay attention to symbols used as well as other small words that are often used. For example, earlier an example was provided on probability …. “the probability of A” and “the probability of A only”. The two expressions are not the same. The small word “only” makes the second expression differ to the first expression.

According to the above extracts, another source of difficulty for learners solving word problems is limited knowledge of the mathematical vocabulary, which according to Krick-Morales (2006), is a requirement to solve word problems successfully. The example cited by Teacher 3 indicates that learners did not have an understanding of the mathematical term “difference”, judging by the answers they provided. The learners understood the meaning of the word “difference” as implied in Sesotho, in order to provide the mathematical answers. In Sesotho the term “difference” implies “phapang” which in English means “unlikeness” between the “objects”, which is what it also implies in Isizulu (e.g. Umehluko), Isixhosa (Umehluko) as well as in “ordinary” English. The learners thus
“compared” the numbers instead of “subtracting” them, to obtain the answer, which is termed “difference” in mathematical context. This clearly shows that learners had limited knowledge of the mathematical vocabulary. In this instance, the learners compared the two numbers, namely, nine and three in order to highlight the difference(s) instead of performing the subtraction operation. The example that was cited by the teacher indicates that learners rely on their home languages in order to make sense of the word problems and this hampers both the conceptual and procedural understandings of the mathematical concepts, which are embedded in a word problem. The statement uttered by Teacher 3, to a certain extent, indicates that teachers do not explicitly teach mathematical vocabulary to promote and advance the understanding of the word problems. In agreement to this finding Fatmanissa and Kusnandi (2017) added that this creates difficulties, as learners rely on vocabulary to make meaning and subsequently solve a problem.

Some solutions were recommended by the participants to address limited knowledge of mathematical vocabulary. Teachers 2 and 4 recommended explicit teaching of mathematical vocabulary. According to Riccomini, Smith, Hughes and Fries (2015, p.241), this form of strategy is called "explicit vocabulary instruction" and it helps learners exercise their vocabulary in mathematics word problems. The two teachers further stressed the fact that teachers should teach learners the application of the terms according to the different contexts, to avoid confusion. In addition, teachers have to make sure that they clarify in their teaching the key terms that are almost similar but have different meanings and implications resulting from the presence of the minor, yet significant adjectives used in mathematics, such as “only”, “mainly”, etc.

An example was provided by Teacher 4 to this effect, namely “the probability of A” and “the probability of A only”. The adjective “only” in the second expression differentiates it from the first expression namely, “probability of A” which does not contain the adjective “only”. According to Teacher 4, the absence of the word “only” in the first expression makes this expression imply what is not implied in the second expression. This therefore means that the first expression necessitates a particular approach in terms of working out (or solving) a problem which will then yield the outcome that is different from the second expression that contains this adjective “only” (Moleko, 2018). In line with this illustration/explanation, it is important for teachers to teach learners in a manner that helps facilitate an understanding of the differences in meaning between key terms that almost “sound and look similar”. This will enable learners to understand the distinction between the key terms, and enable them to apply the correct procedures in solving the related problems. Thus the teachers should encourage learners to read the word problems in detail without overlooking certain words such as “only”, “like”, “is”, etc., which may come across as insignificant, yet could have implications in how the word problem has to be solved (Moleko, 2018).

The language used outside mathematics classroom

The language used outside mathematics classroom was highlighted as one of the possible hindrances towards the teaching and learning of word problems during the focus group discussion. The teachers “painted a picture” as follows:
Teacher 2: When I gave them a problem where the term “at least” was used, I thought they understood its application and know how to represent it but to my surprise, I realised they didn’t. To them, the term “at least” means “bonyane”…

Teacher 5: I also taught learners whose home language was Isizulu previously and they also assigned the same meaning to the key term.

Teacher 1: I think it is important for us to check first if the learners understand the words in context before we proceed without assuming that they know these words and their applications….

Teacher 6: It is indeed important to make sure that learners get the correct interpretation and correctly apply the terms otherwise; they will not be able to illustrate the given information algebraically and diagrammatically.

Figure 1: Correct and incorrect diagrammatic representations of $x \geq 2$

The example cited by Teacher 2 indicates that the language that is used outside classroom, if it is not addressed, has the potential to influence learning in a negative manner. For example; the word “at least” is usually used outside classroom, referring to “small” (for instance “bonyane” in Sesotho, “buncinci” in Isixhosa and “okunani” in Isizulu). The learners apply a similar meaning or use the meaning implied in Sesotho to solve word problems with the term “at least”. These learners usually get incorrect answers because mathematically speaking, the expression “at least” does not imply “small”. The expression “at least” is used predominantly when dealing with the inequality concept in mathematics wherein the following signs are mainly used; $\geq$ or $\leq$. In order to represent the expression “at least” symbolically, the learners use the symbol $\leq$, which is informed by the meaning that is assigned or connoted to this expression outside mathematics context (meaning implied in home languages). Such incorrect application of the mathematical term results in incorrect algebraic representation as well as incorrect diagrammatical representation, as shown in Figure 1.

According to Figure 1, the interpretations that were provided by the learners, to a certain extent indicated that when solving the word problems, learners apply meanings which are used outside classroom, to solve mathematical concepts. To some extent, this indicates that teachers teach without putting much emphasis on distinguishing the use of terms used in mathematics and outside mathematics context. They also do not put more emphasis on illustrating how the expressions are represented symbolically and diagrammatically in line with the mathematical language and structure. This does not
promote learning of the correct application of the various mathematical terms and expressions as well as their different representations.

In order to address the challenges caused by the language used outside mathematics classroom, Teacher 1 advised that the teachers should first recapitulate the meaning of the expression to check whether it is understood by learners within the mathematical context, as opposed to how it is used outside the classroom. This means that the teachers should not assume that learners know the terms and their application. This point was made by Teacher 6 with reference to the application of the expression “at least”. The teacher explained that the expression “at least” is often used outside classroom and that when learners attempt to solve problems related to this expression in class, they usually attach the meaning that is used outside classroom to the expression to solve the related problems. However, this does not help in obtaining the correct solutions since what the expression actually means in mathematical context is different from what it means outside the classroom. For example in Sesotho the expression “at least” is used to imply “bonyane” similarly to Isixhosa (“bucinci”) and Isizulu (“ukunani”), but in these languages the expression implies “little”. However, if a word problem with this expression is provided in mathematical context, then the symbol (≥) denoting greater than or equal to, is used instead of the symbol (≤), which denotes less than or equal to (Moleko, 2018). In this context, one could thus conclude that even though the word “less” can easily be associated with “little”, which is embedded in the expression in the learners’ home languages as well as in ordinary English, its symbolic representation requires the “greater or equal to” (≥) sign to be used. In addition to the teacher’s advice that learners should also be taught the application of the symbols, Adams (2003) added that teaching skills which help learners read mathematics for both its words and its symbols are important, and that teachers should nurture these skills.

Lack of understanding the mathematical language and structure

During one of the reflection sessions, a learner reflected on lack of understanding the mathematical language and structure as one of the challenges pertaining to the teaching and learning of word problems. The learner reflected as follows:

Learner 7: …what I have seen in many instances is that we usually struggle to understand the mathematical language and structure. Let me make an example with one of the problems that was given to us in class to show you how most of us solved that problem… [writing the problem on the board to illustrate how the problem was solved by a learner]

![Figure 2: Learner 7 explanation](image-url)
Learner 8: ...Sometimes it help for us to understand when different parts of the word problems are explained to us separately...

Teacher 4: I find reading and explaining the word problems to my learners helpful. They are able to follow what I am teaching them.

According to Learner 7, most learners do not understand the mathematical language structure. This was made evident by the example that she provided above. From the worked out problem, it was evident that the learners could not distinguish between the two expressions namely, “less than” and “is less than”. The two phrases sound almost similar, yet the difference is made vivid through the latter expression, which has the word “is” in contrast with the former expression, which does not have the word “is”. The learners could not readily understand the two expressions as they did not discern the difference between the two. This means that the learners did not know which of the two expressions necessitated the use of the sign “less than” (\(<\)) and which necessitated the use of a negative sign (\(\text{–}\)) (denoting less than), even though generally speaking, the two signs imply “less”.

Again, from the other worked out problem, Learner 3 expressed “four less than a number” as \(4 – x\) instead of \(x – 4\). The two expressions \(4 – x\) and \(x – 4\) are different. The variable “\(x\)” could be “any number” and if for instance, \(x\) represented a large sum of money which has to be substituted in place of “\(\text{x amount}\)”, then the first expression will give a negative number as an answer (called difference) which may not show the correct picture of the situation in financial context, whereas the second expression will give a positive number (amount) as an answer. The first expression shows that the learner used the “left to right” linear form of reading in trying to write an algebraic expression and thus could not express it algebraically correctly by “swapping around” \(x\) variable and a number 4. Ultimately, the lack of understanding mathematical language structure led to incorrect representation of the whole word problem algebraically.

In order to address lack of understanding mathematical language and structure, the participants recommended that learners be “properly” taught the mathematical language structure. Chitera (2009) shared the same sentiment and avowed that such teaching is necessary because the order of mathematical concepts when expressed in words differs from when the concepts are expressed in symbols. Reading and explaining the word problems while writing on the board was also recommended by Teacher 4 and Learner 5 as one of the strategies that could help improve the mathematical language structure. The strategy could also be used to enable the learners to correctly represent the information. To demonstrate the usefulness of this strategy, Teacher 4 provided the following example:

Teacher 4: A number is greater than three but less than five
The statement can be expressed mathematically in different ways:
It could be represented as: \(x > 3; x < 5\)
Or \(3 < x < 5\)
Of which in both cases it implies: \(3 < x; 5 > x\)
(where “\(x\)” represents “a number”)
For one to be able to represent the above information algebraically, a number line as another form of representation may be used as a viable strategy to guide the correct use of the mathematical structure. The structure of the mathematical expression when phrased in words becomes clearer when a number line is used and this enables the learners to realise the transition from text to algebraic, symbolic and diagrammatical representation (Moleko, 2018). Thus an expression, “x is greater than three but less than five” can be represented symbolically as $3 < x < 5$. Although $3 < x < 5$ and $3 < x; 5 > x$ are comparable, mathematically, $3 < x < 5$ is regarded as the most parsimonious expression which in this context would be the preferred form of representation.

The approach described above is called learner-centred teaching. It is an approach to teaching that is progressively being more encouraged in education (Darsih, 2018). The approach accentuates the use of a variety of different types of methods that shift the role of the teachers from givers of information to facilitators in student learning (Darsih, 2018).

The use of learner-centred pedagogy favours a democratic approach to teaching that shifts the teacher from the centre of the learning environment to a more peripheral position (Moate & Cox, 2015). According to Wright (2011), this shift can be achieved by increasing learners’ opportunities to actively participate in the classroom and engage in self-directed learning outside the classroom, as well as providing opportunities through which they can share learned information with peers. The benefits of this type of teaching are commendable in terms of providing impactful learning experiences (Moate et al., 2015) and for this reason, emphasis should be placed on this mode of teaching delivery as part of teacher professional development. Furthermore, further research must be dedicated to this mode of teaching delivery to “showcase” it as best practice.

**Conclusion**

This present study addresses one of the most important issues in mathematics, namely the teaching of MWPs, which is one of the components of mathematics education that is regarded as challenging. The research question in this study, “how can the teachers’ and learners’ experiences inform the effective teaching and learning of mathematics word problems?” is responded to through this article. It presents the underlying challenges for teaching and learning MWPs as well as providing solutions to these challenges. The study presents experiences from the perspectives of both the teachers and learners, reporting challenges as they experience them and providing solutions to the challenges as they see fit. An emphasis on drawing from the experiences of the learners, and regarding such experiences as important tools for informing the planning of teaching, serves as a best practice highlighted in this study. Thus the study inspires and proposes a “new” idea of learner-centred teaching planning approach, which should be viewed as one important initiative to drive the agenda of inclusive planning of teaching and learning. Overall, findings of this study revealed the significance of teaching mathematical vocabulary since learners rely on it to make meaning and sense of the word problems. The use of home-languages is also deemed important to support learner comprehension of the mathematical concepts. Furthermore, attention must be paid to the application of terms that are used both in mathematics context as well as outside, in order to draw a distinction and thus
avoid confusion. Teachers also need to encourage thorough reading as opposed to scanning, in order for all the words to be taken into account in an endeavour not to lose any meaning.

Drawing from the above, a conclusion can be made that the teaching of MWPs is complex and necessitates efforts from teachers to teach them effectively. Teachers thus need to be aware of the challenges that serve as barriers towards learning MWPs and consequently put measures in place to address such challenges. Of vital significance is the need for teachers to take into consideration the learners’ experiences and incorporate them into their teaching plans. The complexity of teaching MWPs necessitates further strengthening of teacher professional development, to improve enlightenment about new and better ways to teach this mathematical genre.

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References


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